Channel Aware Distributed Detection in Wireless Network with Correlated Observations

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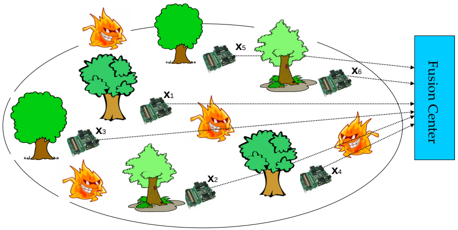
Centralized versus Distributed Detection



fire detection in forest via wireless sensor network

- Centralized detection:
  - Unlimited energy and bandwidth ⇒ infinite precision for sending observations.
  - Error-free communication channels.
- Distributed detection:
  - Passing local decisions to the FC.
    - Classical: error-free communication channels.
    - Our model: fading and noise in communication channels.
- Design of distributed detection system.

The Problem and Our Approach

Problem 1(P1)

What can be the new architectures for the distributed detection system design in the presence of fading and noise in communication channels?

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Our Approach

We propose three new architectures:

- (i) cooperative fusion architecture with Alamouti's STC scheme at sensors,
- (ii) cooperative fusion architecture with signal fusion at sensors,
- (iii) parallel fusion architecture with local threshold changing at sensors.

The Problem and Our Approach

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For distributed detection of a Gaussian signal source in noise, what is the optimal transmit power allocation at sensors?

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Our Approach

For linear fusion rule at the FC and

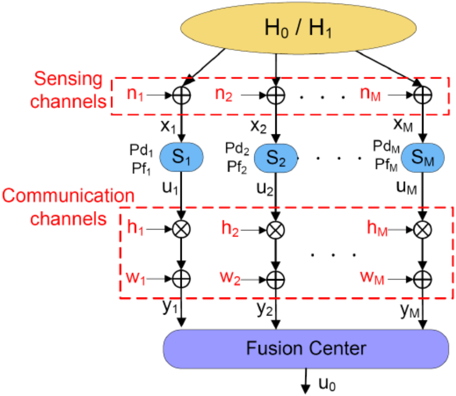
- Total or individual transmit power constraints at sensors,
- Coherent and noncoherent reception mode at the FC,
- Different communication multiple access channel schemes.

We find transmit power allocation at sensors, such that modified deflection coefficient (MDC) at FC is maximized.

Distributed Binary Detection over Fading Channels: Cooperative and Parallel Architectures

Parallel Fusion Architecture

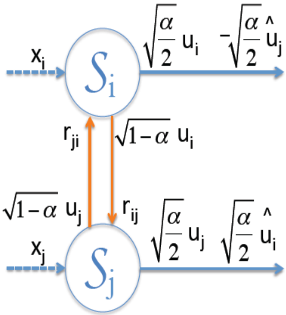
- <u>Sensing Channel Model</u>
  - $\mathcal{H}_0 : x_k = w_k; \mathcal{H}_1 : x_k = 1 + w_k;$ $w_k \sim \mathcal{N}(0, \sigma^2_{w_k})$.
  - $\mathcal{S}_k$ applies the LRT,
    $$\frac{f(x_k|\mathcal{H}_1)}{f(x_k|\mathcal{H}_0)} \underset{u_k=\text{-}1}{\overset{u_k=1}{\gtrless}} \frac{\pi_0}{\pi_1}.$$
  - $P_{d_k} = P(u_k = 1|\mathcal{H}_1)$ and $P_{f_k} = P(u_k = 1|\mathcal{H}_0)$.
- <u>Communication Channel Model</u>
  - $y_k = u_k h_k + v_k; h_k \sim \mathcal{CN}(0, \sigma^2_{h_k}), \quad v_k \sim \mathcal{CN}(0, \sigma^2_v)$.
  - The FC forms the LRT,
    $$\Lambda = \frac{f(y_1, \ldots, y_K|\mathcal{H}_1)}{f(y_1, \ldots, y_K|\mathcal{H}_0)} \underset{U_0=0}{\overset{U_0=1}{\gtrless}} \frac{\pi_0}{\pi_1}.$$
  - If w_ks are uncorrelated, we have
    $$\Lambda = \prod_{k=1}^{K} \frac{P_{d_k} f(y_k|u_k=1) + (1-P_{d_k}) f(y_k|u_k=-1)}{P_{f_k} f(y_k|u_k=1) + (1-P_{f_k}) f(y_k|u_k=-1)}.$$



Cooperative Fusion Architecture with STC at Sensors

Sensing Channel Model

- $\mathcal{S}_i$ and $\mathcal{S}_j$ are cooperative partners.
- $\mathcal{S}_i$ transmits $\sqrt{1-\alpha}u_i$, where $0 < \alpha < 1$.
- $r_{ij} = \sqrt{1-\alpha}u_i g_{ij} + \eta_{ij}$,
  $g_{ij} \sim \mathcal{CN}(0, \sigma^2_{hs_{ij}})$, $\eta_{ij} \sim \mathcal{CN}(0, \sigma^2_\eta)$.
- $\mathcal{S}_j$ demodulates u_i, using the knowledge of g_{ij},
  $\hat{u}_i = sgn(Re(r_{ij}/g_{ij}))$.
- nth slot: $\mathcal{S}_i$ and $\mathcal{S}_j$ send $\sqrt{\frac{\alpha}{2}}u_i$ and $\sqrt{\frac{\alpha}{2}}u_j$.
  $(n+1)$th slot: $\mathcal{S}_i$ and $\mathcal{S}_j$ send $-\sqrt{\frac{\alpha}{2}}\hat{u}_j$ and $\sqrt{\frac{\alpha}{2}}\hat{u}_i$.



Cooperative Fusion Architecture with STC at Sensors

Communication Channel Model

- We have

$$y_{ij}(n) = \sqrt{\frac{\alpha}{2}}(u_i h_i + u_j h_j) + v_{ij}(n), \;\; y_{ij}(n+1) = \sqrt{\frac{\alpha}{2}}(\hat{u}_i h_j - \hat{u}_j h_i) + v_{ij}(n+1)$$
$$h_i \sim \mathcal{CN}(0, \sigma^2_{h_i}), h_j \sim \mathcal{CN}(0, \sigma^2_{h_j}), v_{ij}(n), v_{ij}(n+1) \sim \mathcal{CN}(0, \sigma^2_v).$$

- The FC forms

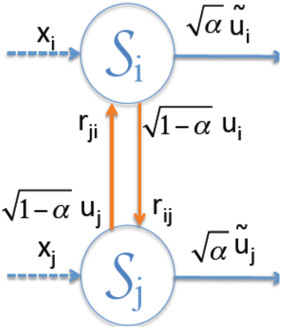
$$\begin{bmatrix} z_i \\ z_j \end{bmatrix} = \begin{bmatrix} h_i^* & h_j \\ h_j^* & -h_i \end{bmatrix} \begin{bmatrix} y_{ij}(n) \\ y_{ij}^*(n+1) \end{bmatrix} = \begin{bmatrix} h_i^* & h_j \\ h_j^* & -h_i \end{bmatrix} \begin{bmatrix} v_{ij}(n) \\ v_{ij}^*(n+1) \end{bmatrix}$$
$$+ \sqrt{\frac{\alpha}{2}} \left(\begin{bmatrix} |h_i|^2 & h_j h_i^* \\ h_i h_j^* & |h_j|^2 \end{bmatrix} \begin{bmatrix} u_i \\ u_j \end{bmatrix} + \begin{bmatrix} |h_j|^2 & -h_j h_i^* \\ -h_i h_j^* & |h_i|^2 \end{bmatrix} \begin{bmatrix} \hat{u}_i \\ \hat{u}_j \end{bmatrix} \right).$$

- Using the h_i, h_j for all pairs, the FC forms LRT $\Lambda = \frac{f(z_i, z_j \text{ for all pairs} | \mathcal{H}_1)}{f(z_i, z_j \text{ for all pairs} | \mathcal{H}_0)} \underset{U_0=0}{\overset{U_0=1}{\gtrless}} \frac{\pi_0}{\pi_1}$.

Cooperative Fusion Architecture with Signal Fusion at Sensors

Sensing Channel Model

- $\mathcal{S}_j$ updates its initial decision by fusing r_{ij} and x_j and forms $\tilde{\lambda}_j = \frac{f(r_{ij}, x_j | \mathcal{H}_1)}{f(r_{ij}, x_j | \mathcal{H}_0)} \underset{\tilde{u}_j=\text{-}1}{\overset{\tilde{u}_j=1}{\gtrless}} \frac{\pi_0}{\pi_1}$.
- The pair $(\mathcal{S}_i, \mathcal{S}_j)$ sends $\sqrt{\alpha}\tilde{u}_i, \sqrt{\alpha}\tilde{u}_j$ to the FC over two orthogonal channels subject to noise and fading.



Communication Channel Model

- We have

$$y_i = \sqrt{\alpha}\tilde{u}_i h_i + v_i, \quad y_j = \sqrt{\alpha}\tilde{u}_j h_j + v_j,$$
$$h_i \sim \mathcal{CN}(0, \sigma_{h_i}^2), \quad h_j \sim \mathcal{CN}(0, \sigma_{h_j}^2), \quad v_i, v_j \sim \mathcal{CN}(0, \sigma_v^2).$$

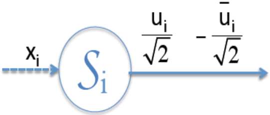
- Using h_i, h_j for all pairs, the FC forms the LRT $\Lambda = \frac{f(y_i, y_j \text{ for all pairs} | \mathcal{H}_1)}{f(y_i, y_j \text{ for all pairs} | \mathcal{H}_0)} \underset{U_0=0}{\overset{U_0=1}{\gtrless}} \frac{\pi_0}{\pi_1}$, to make the final decision.

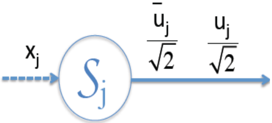
Parallel Fusion Architecture with Local Threshold Changing at Sensors

Sensing Channel Model

- In the absence of inter-node communication, $\mathcal{S}_i$ assumes $u_j = -u_i$.
- $\mathcal{S}_i$ forms $\bar{u}_i$ by fusing the assumed decision u_j and x_i.
  $$\bar{\lambda}_i = \frac{f(x_i, u_j = -u_i | \mathcal{H}_1)}{f(x_i, u_j = -u_i | \mathcal{H}_0)} \underset{\bar{u}_i=-1}{\overset{\bar{u}_i=1}{\gtrless}} .$$
- One can verify that
  $$u_i = 1,\ \bar{u}_i = 1 \text{ if } x_i > \tau'_{i_1},\quad u_i = -1,\ \bar{u}_i = -1 \text{ if } x_i < \tau'_{i_2},$$
  $$u_i = -1,\ \bar{u}_i = 1 \text{ if } \tau'_{i_2} < x_i < \tau_i,$$
  $$u_i = 1,\ \bar{u}_i = -1 \text{ if } \tau_i < x_i < \tau'_{i_1}$$

  where the thresholds τ'_{i_1}, τ'_{i_2} depend on $\sigma^2_{w_i}, \rho_{i,j}$ and satisfy $\tau'_{i_2} < \tau_i < \tau'_{i_1}$.





Performance Analysis

Assumptions

- Gaussian sensing noises w_k are i.i.d. thus $P_{d_k} = P_d, P_{f_k} = P_f$.
- Sensors are positioned equally distant from the FC and thus $\bar{\gamma}_h^2 = \frac{\sigma_h^2}{\sigma_v^2}$.
- Distances between the cooperative partners are assumed equal across the pairs and therefore $\bar{\gamma}_{hs}^2 = \frac{(1-\alpha)\sigma_{hs}^2}{\sigma_\eta^2}$.

Parallel Fusion Architecture

$$\bar{P}_{e_1} = \pi_0 \sum_n \bar{\mathcal{T}}_{e_1} P_f^{Q_n} (1 - P_f)^{K-Q_n}$$

$$\bar{P}_{e_2} = \pi_1 \sum_n \bar{\mathcal{T}}_{e_2} P_d^{Q_n} (1 - P_d)^{K-Q_n}$$

$$\bar{\mathcal{T}}_{e_1} < \frac{\mathbf{1}_{\{Q_n<M\}}}{2\sqrt{|S_1|}} \sum_{d_{n_1} \in S_1} [\sqrt{G(n, n_1)} \prod_{s=1}^{S} \mathcal{D}_1(n, n_1)] + \mathbf{1}_{\{Q_n \geq M\}},$$

$$\bar{\mathcal{T}}_{e_2} < \frac{\mathbf{1}_{\{Q_n>M\}}}{|S_0|} \sum_{d_{n_1} \in S_0} [\min_t (|S_0| G(n, n_1))^t \prod_{s=1}^{S} \mathcal{D}_2(n, n_1)] + \mathbf{1}_{\{Q_n \leq M\}},$$

$$\mathcal{D}_1(n, n_1) = \left((1 + \frac{\bar{\gamma}_h |a_n^{2s-1} - a_{n_1}^{2s-1}|}{2})(1 + \frac{\bar{\gamma}_h |a_n^{2s} - a_{n_1}^{2s}|}{2}) \right)^{-1},$$

$$\mathcal{D}_2(n, n_1) = \left((1 + 2(t^2 - t)\bar{\gamma}_h |a_n^{2s-1} - a_{n_1}^{2s-1}|)(1 + 2(t^2 - t)\bar{\gamma}_h |a_n^{2s} - a_{n_1}^{2s}|) \right)^{-1}.$$

Cooperative Fusion Architecture with STC at Sensors

$$\bar{P}_{e_1} = \pi_0 \sum_{n,m} \bar{\mathcal{T}}_{e_1} P_f^{Q_n} (1 - P_f)^{K - Q_n} T_{n,m}$$

$$\bar{P}_{e_2} = \pi_1 \sum_{n,m} \bar{\mathcal{T}}_{e_2} P_d^{Q_n} (1 - P_d)^{K - Q_n} T_{n,m}.$$

$$\bar{\mathcal{T}}_{e_1} < \frac{\mathbf{1}_{\{Q_n < M\}}}{2\sqrt{|S_1|}} \sum_{d_{n_1,m_1} \in S_1} [\sqrt{G(n,m,n_1,m_1)} \prod_{s=1}^{S} \mathcal{D}_1(n,m,n_1,m_1)] + \mathbf{1}_{\{Q_n \geq M\}}$$

$$\bar{\mathcal{T}}_{e_2} < \frac{\mathbf{1}_{\{Q_n > M\}}}{|S_0|} \sum_{d_{n_1,m_1} \in S_0} [\min_t (|S_0| G(n,m,n_1,m_1))^t \prod_{s=1}^{S} \mathcal{D}_2(n,m,n_1,m_1)] + \mathbf{1}_{\{Q_n \leq M\}},$$

$$\mathcal{D}_1(n,m,n_1,m_1) = \left((1 + \frac{\alpha \bar{\gamma}_h \bar{a}_1}{8})(1 + \frac{\alpha \bar{\gamma}_h \bar{a}_2}{8}) - \frac{\alpha^2 \bar{\gamma}_h^2 \bar{a}_3}{64} \right)^{-1},$$

$$\mathcal{D}_2(n,m,n_1,m_1) = \left((1 + \frac{\alpha (t^2 - t) \bar{\gamma}_h \bar{a}_1}{2})(1 + \frac{\alpha (t^2 - t) \bar{\gamma}_h \bar{a}_2}{2}) - \frac{\alpha^2 (t^2 - t)^2 \bar{\gamma}_h^2 \bar{a}_3}{16} \right)^{-1}$$

When inter-sensor channels are error-free $\bar{a}_3 = 0$ and when $\bar{\gamma}_h$ is high we have

$$\mathcal{D}_1(.) = (\frac{\alpha \bar{a}_1}{8} \frac{\alpha \bar{a}_2}{8})^{-1} \bar{\gamma}_h^{-2 \rightarrow \text{diversity gain}}, \mathcal{D}_2(.) = (\frac{\alpha (t^2 - t) \bar{a}_1}{2} \frac{\alpha (t^2 - t) \bar{a}_2}{2})^{-1} \bar{\gamma}_h^{-2 \rightarrow \text{diversity gain}}$$

Cooperative Fusion Architecture with Signal Fusion at Sensors

$$\bar{P}_{e_1} = \pi_0 \sum_n \bar{\mathcal{T}}_{e_1} P_f^{Q_n} (1 - P_f)^{K - Q_n}$$

$$\bar{P}_{e_2} = \pi_1 \sum_n \bar{\mathcal{T}}_{e_2} P_d^{Q_n} (1 - P_d)^{K - Q_n}$$

$$\bar{\mathcal{T}}_{e_1} < \frac{\mathbf{1}_{\{d_n \in S_0\}}}{2\sqrt{|S_1|}} \sum_{d_{n_1} \in S_1} [\sqrt{G(n, n_1)} \prod_{s=1}^{S} \mathcal{D}_1(n, n_1)] + \mathbf{1}_{\{d_n \in S_1\}} \rightarrow \text{making local decision more reliable},$$

$$\bar{\mathcal{T}}_{e_2} < \frac{\mathbf{1}_{\{d_n \in S_1\}}}{|S_0|} \sum_{d_{n_1} \in S_1} [\min_t (|S_0| G(n, n_1))^t \prod_{s=1}^{S} \mathcal{D}_2(n, n_1)] + \mathbf{1}_{\{d_n \in S_0\}} \rightarrow \text{making local decision more reliable},$$

$$\mathcal{D}_1(n, n_1) = \left((1 + \frac{\alpha \bar{\gamma}_h (a_n^{2s-1} - a_{n_1}^{2s-1})^2}{4})(1 + \frac{\alpha \bar{\gamma}_h (a_n^{2s} - a_{n_1}^{2s})^2}{4}) \right)^{-1} \rightarrow \text{no diversity gain},$$

$$\mathcal{D}_2(n, n_1) = \left((1 + \alpha(t^2 - t)\bar{\gamma}_h (a_n^{2s-1} - a_{n_1}^{2s-1})^2)(1 + \alpha(t^2 - t)\bar{\gamma}_h (a_n^{2s} - a_{n_1}^{2s})^2) \right)^{-1} \rightarrow \text{no diversity gain}.$$

Parallel Fusion Architecture with Local Threshold Changing at Sensors

$$\bar{P}_{e_1} = \pi_0 \sum_{n,m} \bar{\mathcal{T}}_{e_1} \prod_{j=1}^{4} P_{f_j}^{Q^j_{n_1,m_1}} .$$

$$\bar{P}_{e_2} = \pi_1 \sum_{n,m} \bar{\mathcal{T}}_{e_2} \prod_{j=1}^{4} P_{d_j}^{Q^j_{n_1,m_1}} .$$

$$\bar{\mathcal{T}}_{e_1} < \frac{\mathbf{1}_{\{d_{n,m} \in S_0\}}}{2\sqrt{|S_1|}} \sum_{d'_{n_1,m_1} \in S_1} [\sqrt{G(n,m,n_1,m_1)} \prod_{s=1}^{S} \mathcal{D}_1(n,m,n_1,m_1)] + \mathbf{1}_{\{d_{n,m} \in S_1\}},$$

$$\bar{\mathcal{T}}_{e_2} < \frac{\mathbf{1}_{\{d_{n,m} \in S_1\}}}{|S_0|} \sum_{d_{n,m} \in S_0} [\min_t (|S_0| G(n,m,n_1,m_1))^t \prod_{s=1}^{S} \mathcal{D}_2(n,m,n_1,m_1)] + \mathbf{1}_{\{d_{n,m} \in S_0\}},$$

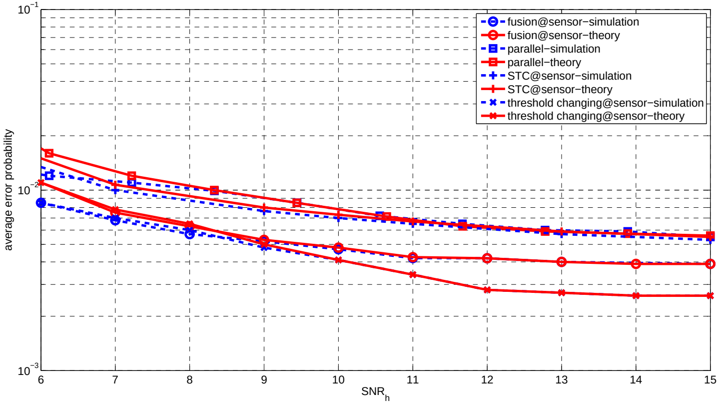
$$\mathcal{D}_1(n,m,n_1,m_1) = \left((1 + \frac{\alpha \bar{\gamma}_h \bar{a}_1}{8})(1 + \frac{\alpha \bar{\gamma}_h \bar{a}_2}{8}) - \frac{\alpha^2 \bar{\gamma}_h^2 \bar{a}_3}{64} \right)^{-1},$$

$$\mathcal{D}_2(n,m,n_1,m_1) = \left((1 + \frac{\alpha(t^2 - t)\bar{\gamma}_h \bar{a}_1}{2})(1 + \frac{\alpha(t^2 - t)\bar{\gamma}_h \bar{a}_2}{2}) - \frac{\alpha^2 (t^2 - t)^2 \bar{\gamma}_h^2 \bar{a}_3}{16} \right)^{-1}.$$

Numerical Results Setup

- $K=10$, $\sigma^2_{w_k}=\sigma^2_w$, $\rho_{ij}=\rho$, $\text{SNR}_c=-20\log_{10}\sigma_w$, $d=10m$, $d_0=2m$.
- In "STC@sensors" and "fusion@sensors" a sensor spends $(1-\alpha)\mathcal{P}$ and $\alpha\mathcal{P}$, respectively, for communicating with its cooperative partner and with the FC, where α is different in these two schemes.
- $\text{SNR}_h = 10\log_{10}\bar{\gamma}_h$, in which $\bar{\gamma}_h=\frac{\sigma_h^2}{\sigma_v^2}=\frac{\mathcal{P}\mathcal{G}}{d^\varepsilon\sigma_v^2}$, $\sigma_v^2=\sigma_\eta^2=-50dBm$, $\varepsilon=2$, $\mathcal{G}=-30\text{dB}$.

Numerical Results



Monte-Carlo simulation versus analytical results

Numerical Results

- "STC@sensors" versus "parallel":
  - Moderate SNR_h and moderate/high SNR_c: "STC@sensors" $>$ "parallel".
  - Otherwise: "parallel" $>$ "STC@sensors".
- "fusion@sensors" versus "parallel":
  - Low SNR_h: "fusion@sensors" $\approx$ "parallel".
  - Moderate/high SNR_h: "fusion@sensors" $>$ "parallel".
- "threshold changing@sensors" versus "parallel":
  - Moderate/high SNR_h: "threshold changing@sensors" $>$ "parallel".
  - Low SNR_h and low SNR_c: "threshold changing@sensors" $>$ "parallel".
  - Otherwise: "parallel" $>$ "threshold changing@sensors".

Numerical Results

- In general
  - Moderate/high SNR_h: “threshold changing@sensors” $>$ others.
  - Low SNR_h and low SNR_c: “threshold changing@sensors” $>$ others.
  - Low SNR_h and moderate/high SNR_c: “fusion@sensors” $\approx$ “parallel” $>$ others.
- “STC@sensors” improves $\bar{P}e$ by via providing diversity gain.
  1. Diversity gain is achieved only in moderate/high SNRs.
  2. “STC@sensors” and “parallel” have the same error floor.

  - 1 and 2 $\Rightarrow$ “STC@sensors” > “parallel” only at moderate SNR_h.
- “fusion@sensors” improves $\bar{P}e$ by increasing the reliability of local decision.
  - “parallel” $>$ “fusion@sensors” at low SNR_h because $\bar{P}e$ is governed by communication channel.
- The above findings on comparison between different architectures remain the same in asymptotic regime when $K \to \infty$.

Numerical Results-Impact of Correlation on Performance Comparison

- $\rho \approx 0.2 - 0.3$:
  - High SNR$_h$: "threshold changing@sensors" $>$ others.
  - Medium SNR$_h$: "fusion@sensors" $>$ others.
  - Low SNR$_h$: "parallel" and "fusion@sensor" $>$ others.
- $\rho = 0.5$:
  - High SNR$_h$ and high SNR$_c$: "threshold changing@sensors" $>$ others.
  - High SNR$_h$ and medium/low SNR$_c$ and for medium SNR$_h$: "fusion@sensors" $>$ others.
  - Low SNR$_h$: "parallel" and "fusion@sensor" $>$ others.
- $\rho = 0.8$:
  - "threshold changing@sensors" $<$ others.

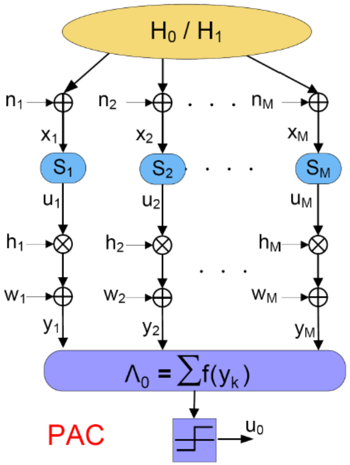
Deflection-Optimal Power Allocation for Distributed Detection with Correlated Observations and Linear Fusion

System Model and Problem Statement

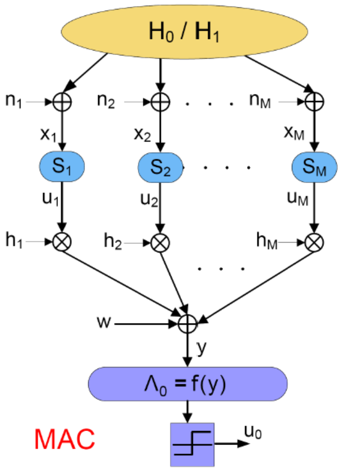
- Sensing Channel Model
  - $\mathcal{H}_0 : \boldsymbol{x} \sim \mathcal{N}(0, \sigma_0 \boldsymbol{I}), \quad \mathcal{H}_1 : \boldsymbol{x} \sim \mathcal{N}(0, \boldsymbol{\Sigma})$.
    σ_0 is the variance under $\mathcal{H}_0$ and $\boldsymbol{\Sigma}$ is a non-diagonal positive definite covariance matrix under $\mathcal{H}_1$,
    i.e., under $\mathcal{H}_1$ ($\mathcal{H}_0$) sensors' observations are correlated (uncorrelated) Gaussian.
- Communication Channel Model
  - u_k is communicated to the FC with transmit power $\mathcal{P}_{t_k}$. Let $h_k = |h_k| e^{j\phi_k}$. We have
    - Coherent PAC : $y_k = \sqrt{\mathcal{P}_k} |h_k| u_k + n_k$,
    - Noncoherent PAC : $y_k = \sqrt{\mathcal{P}_k} h_k u_k + n_k$,
    - Coherent MAC : $y = \sum_{k=1}^{M} \sqrt{\mathcal{P}_k} |h_k| u_k + n$,
    - Noncoherent MAC : $y = \sum_{k=1}^{M} \sqrt{\mathcal{P}_k} h_k u_k + n$.

    $n_k \sim \mathcal{CN}(0, \sigma_n^2)$, $n \sim \mathcal{CN}(0, \sigma_n^2)$.
    Also $\mathcal{P}_k = \mathcal{P}_{t_k} \theta_k$, $\theta_k = G d_{FS_k}^{-\epsilon_c}$.

PAC vs MAC



H0 / H1
n1
n2
nM
x1
x2
xM
S1
S2
SM
u1
u2
uM
h1
h2
hM
w1
w2
wM
y1
y2
yM
Λ0 = Σf(yk)
u0
PAC



H0 / H1
n1
n2
nM
x1
x2
xM
S1
S2
SM
u1
u2
uM
h1
h2
hM
w
y
Λ0 = f(y)
u0
MAC

System Model and Problem Statement

- $T \underset{U_0=0}{\overset{U_0=1}{\gtrless}} \tau_0$. We let the fusion statistic T be
  - Coherent PAC : $T = \sum_{k=1}^{M} \text{Re}(y_k)$,
    Noncoherent PAC : $T = \sum_{k=1}^{M} |y_k|^2$,
  - Coherent MAC : $T = \text{Re}(y)$, Noncoherent MAC : $T = |y|^2$.
- Depending on the availability of CSI at the FC, we have
  - Full CSI at the FC,
  - Knowledge of channel statistics at the FC.

Our Goal:

Aiming to maximize MDF,

$$\text{MDF}(T) = \frac{\left(\mathbb{E}(T|\mathcal{H}_1) - \mathbb{E}(T|\mathcal{H}_0)\right)^2}{\text{var}(T|\mathcal{H}_1)}$$

Find the optimal power allocation between sensors under total and individual constraints on their transmit power.

Deriving Modified Deflection Coefficient

$$\text{coherent} : \text{MDC}(\boldsymbol{a}_t) = \frac{\boldsymbol{a}_t^T \boldsymbol{b}_t \boldsymbol{b}_t^T \boldsymbol{a}_t}{\boldsymbol{a}_t^T \boldsymbol{K}_t \boldsymbol{a}_t + c},$$

$$\text{noncoherent} : \text{MDC}(\boldsymbol{\mathcal{P}}_t) = \frac{\boldsymbol{\mathcal{P}}_t^T \boldsymbol{b}_t \boldsymbol{b}_t^T \boldsymbol{\mathcal{P}}_t}{\boldsymbol{\mathcal{P}}_t^T \boldsymbol{K}_t \boldsymbol{\mathcal{P}}_t + \boldsymbol{\mathcal{P}}_t^T \boldsymbol{d}_t + c}.$$

- $\mathcal{P}_k = \mathcal{P}_{t_k}\theta_k$, $a_{t_k} = \sqrt{\mathcal{P}_{t_k}} = \frac{a_k}{\sqrt{\theta_k}}$, $\boldsymbol{a}_t = [a_{t_1}, ..., a_{t_M}]^T$, $\boldsymbol{\mathcal{P}}_t = [\mathcal{P}_{t_1}, ..., \mathcal{P}_{t_M}]^T$, $\boldsymbol{\Theta} = \text{DIAG}\{[\theta_1, ..., \theta_M]^T\}$.
- $\boldsymbol{b}_t$ and $\boldsymbol{K}_t$ in MDC($\boldsymbol{a}_t$) are identical for PAC and MAC.
- c in MDC($\boldsymbol{a}_t$) is M times larger in PAC.
- $\boldsymbol{b}_t$ and $\boldsymbol{d}_t$ in MDC($\boldsymbol{\mathcal{P}}_t$) are identical for PAC and MAC.
- $\boldsymbol{K}_t$ in MDC($\boldsymbol{\mathcal{P}}_t$) are different for PAC and MAC.
- c in MDC($\boldsymbol{\mathcal{P}}_t$) is M times larger in PAC.

Deriving Modified Deflection Coefficient

The three sets of constraints are:

- (A) TPC: $\boldsymbol{a}_t^T \boldsymbol{a}_t \leq \mathcal{P}_{tot}$ for coherent and $\mathbf{1}^T \boldsymbol{\mathcal{P}}_t \leq \mathcal{P}_{tot}$ for noncoherent;
- (B) IPC: $\mathbf{0} \preceq \boldsymbol{a}_t \preceq \sqrt{\boldsymbol{\mathcal{P}}_0}$ for coherent and $\mathbf{0} \preceq \boldsymbol{\mathcal{P}}_t \preceq \boldsymbol{\mathcal{P}}_0$ for noncoherent where $\boldsymbol{\mathcal{P}}_0 = [\mathcal{P}_{0_1}, ..., \mathcal{P}_{0_M}]^T$.
- (C) TIPC: Both TPC and IPC.

Maximizing MDC under TPC

$$\begin{aligned} \max_{\boldsymbol{a}_t} . \quad & \text{MDC}(\boldsymbol{a}_t) = \frac{\boldsymbol{a}_t^T \boldsymbol{b}_t \boldsymbol{b}_t^T \boldsymbol{a}_t}{\boldsymbol{a}_t^T \boldsymbol{K}_t \boldsymbol{a}_t + c} \quad (\mathcal{O}_1) \\ \text{s.t.} \quad & \boldsymbol{a}_t^T \boldsymbol{a}_t \leq \mathcal{P}_{tot} \\ & \boldsymbol{a}_t \succeq \boldsymbol{0} \end{aligned} \quad , \quad \begin{aligned} \max_{\boldsymbol{\mathcal{P}}_t} . \quad & \text{MDC}(\boldsymbol{\mathcal{P}}_t) = \frac{\boldsymbol{\mathcal{P}}_t^T \boldsymbol{b}_t \boldsymbol{b}_t^T \boldsymbol{\mathcal{P}}_t}{\boldsymbol{\mathcal{P}}_t^T \mathbf{K}_t \boldsymbol{\mathcal{P}}_t + \boldsymbol{\mathcal{P}}_t^T \mathbf{d}_t + c} \quad (\mathcal{O}_2) \\ \text{s.t.} \quad & \mathbf{1}^T \boldsymbol{\mathcal{P}}_t \leq \mathcal{P}_{tot} \\ & \boldsymbol{\mathcal{P}}_t \succeq \mathbf{0} \end{aligned}$$

Maximizing MDC under TPC

$$\max_{\boldsymbol{a}_t}.\quad \mathrm{MDC}(\boldsymbol{a}_t) = \frac{\boldsymbol{a}_t^T \boldsymbol{b}_t \boldsymbol{b}_t^T \boldsymbol{a}_t}{\boldsymbol{a}_t^T \boldsymbol{K}_t \boldsymbol{a}_t + c} \quad (\mathcal{O}_1) \qquad \text{s.t.} \quad \boldsymbol{a}_t^T \boldsymbol{a}_t = \mathcal{P}_{tot},\ \boldsymbol{a}_t \succeq \boldsymbol{0}$$

,

$$\max_{\boldsymbol{\mathcal{P}}_t}.\quad \mathrm{MDC}(\boldsymbol{\mathcal{P}}_t) = \frac{\boldsymbol{\mathcal{P}}_t^T \boldsymbol{b}_t \boldsymbol{b}_t^T \boldsymbol{\mathcal{P}}_t}{\boldsymbol{\mathcal{P}}_t^T \boldsymbol{K}_t \boldsymbol{\mathcal{P}}_t + \boldsymbol{\mathcal{P}}_t^T \boldsymbol{d}_t + c} \quad (\mathcal{O}_2) \qquad \text{s.t.} \quad \boldsymbol{1}^T \boldsymbol{\mathcal{P}}_t = \mathcal{P}_{tot},\ \boldsymbol{\mathcal{P}}_t \succeq \boldsymbol{0}$$

- $(\mathcal{O}_1)$: $\hat{\boldsymbol{q}} = \frac{\boldsymbol{q}}{||\boldsymbol{q}||}$ where $\boldsymbol{q} = \boldsymbol{Q}_1^{-1} \boldsymbol{b}_t$, $\boldsymbol{Q}_1 = \boldsymbol{K}_t + \frac{c}{\mathcal{P}_{tot}} \boldsymbol{I}$.
  $\hat{\boldsymbol{q}} \succeq \boldsymbol{0}$: $\boldsymbol{a}_t^* = \hat{\boldsymbol{q}} \sqrt{\mathcal{P}_{tot}}$, $-\hat{\boldsymbol{q}} \succeq \boldsymbol{0}$: $\boldsymbol{a}_t^* = -\hat{\boldsymbol{q}} \sqrt{\mathcal{P}_{tot}}$.
- $(\mathcal{O}_2)$:
  - $\boldsymbol{Q}_2 \succ \boldsymbol{0}$: $\hat{\boldsymbol{q}} = \frac{\boldsymbol{q}}{||\boldsymbol{q}||}$ where $\boldsymbol{q} = \boldsymbol{Q}_2^{-1} \boldsymbol{b}_t$, $\boldsymbol{Q}_2 = \boldsymbol{K}_t + \frac{(\boldsymbol{d}_t \boldsymbol{1}^T + \boldsymbol{1} \boldsymbol{d}_t^T)}{2\mathcal{P}_{tot}} + \frac{c}{\mathcal{P}_{tot}^2} \boldsymbol{1}\boldsymbol{1}^T$.
    $\hat{\boldsymbol{q}} \succeq \boldsymbol{0}$ or $-\hat{\boldsymbol{q}} \succeq \boldsymbol{0}$: $\boldsymbol{\mathcal{P}}_t^* = \frac{\hat{\boldsymbol{q}}}{\boldsymbol{1}^T \hat{\boldsymbol{q}}} \mathcal{P}_{tot}$.
  - $\boldsymbol{Q}_2 \prec \boldsymbol{0}$: we turn $(\mathcal{O}_2)$ to a SDP problem and find an approximate numerical solution.
- All the entries $\hat{\boldsymbol{q}}$ do not have the same sign: we turn $(\mathcal{O}_1)$ and $(\mathcal{O}_2)$ into convex problems and solve them numerically.

Maximizing MDC under TIPC

$$\begin{array}{lcl} \max\limits_{\boldsymbol{a}_t} . & \frac{\boldsymbol{a}_t^T \boldsymbol{b}_t \boldsymbol{b}_t^T \boldsymbol{a}_t}{\boldsymbol{a}_t^T \boldsymbol{K}_t \boldsymbol{a}_t + c} & (\mathcal{O}_3) \\ \text{s.t.} & \boldsymbol{a}_t^T \boldsymbol{a}_t \leq \mathcal{P}_{tot} & \\ & \boldsymbol{0} \preceq \boldsymbol{a}_t \preceq \sqrt{\boldsymbol{\mathcal{P}}_0} & \end{array} , \quad \begin{array}{lcl} \max\limits_{\boldsymbol{\mathcal{P}}_t} . & \frac{\boldsymbol{\mathcal{P}}_t^T \boldsymbol{b}_t \boldsymbol{b}_t^T \boldsymbol{\mathcal{P}}_t}{\boldsymbol{\mathcal{P}}_t^T \boldsymbol{K}_t \boldsymbol{\mathcal{P}}_t + \boldsymbol{\mathcal{P}}_t^T \boldsymbol{d}_t + c} & (\mathcal{O}_4) \\ \text{s.t.} & \boldsymbol{1}^T \boldsymbol{\mathcal{P}}_t \leq \mathcal{P}_{tot} & \\ & \boldsymbol{0} \preceq \boldsymbol{\mathcal{P}}_t \preceq \boldsymbol{\mathcal{P}}_0 & \end{array}$$

Maximizing MDC under TIPC

$$\max_{\boldsymbol{a}_t} . \quad \frac{\boldsymbol{a}_t^T \boldsymbol{b}_t \boldsymbol{b}_t^T \boldsymbol{a}_t}{\boldsymbol{a}_t^T \boldsymbol{K}_t \boldsymbol{a}_t + c} \quad (\mathcal{O}_3') \qquad \text{s.t.} \quad \boldsymbol{a}_t^T \boldsymbol{a}_t = \mathcal{P}_{tot}, \quad \boldsymbol{0} \preceq \boldsymbol{a}_t \preceq \sqrt{\boldsymbol{\mathcal{P}}_0}$$

$$\max_{\boldsymbol{\mathcal{P}}_t} . \quad \frac{\boldsymbol{\mathcal{P}}_t^T \boldsymbol{b}_t \boldsymbol{b}_t^T \boldsymbol{\mathcal{P}}_t}{\boldsymbol{\mathcal{P}}_t^T \boldsymbol{K}_t \boldsymbol{\mathcal{P}}_t + \boldsymbol{\mathcal{P}}_t^T \boldsymbol{d}_t + c} \quad (\mathcal{O}_4') \qquad \text{s.t.} \quad \boldsymbol{1}^T \boldsymbol{\mathcal{P}}_t = \mathcal{P}_{tot}, \quad \boldsymbol{0} \preceq \boldsymbol{\mathcal{P}}_t \preceq \boldsymbol{\mathcal{P}}_0$$

- We first obtain the corresponding TPC solution, $\boldsymbol{a}_{t1}^*$ and $\boldsymbol{\mathcal{P}}_{t1}^*$.
  If the solution does not satisfy the box constraints then the closest point to the solution that satisfies the box constraints is the solution.

$$\min_{\boldsymbol{a}_t} . \quad |\boldsymbol{a}_t - \boldsymbol{a}_{t1}^*|^2 \quad (\mathcal{O}_3'') \qquad \text{s.t.} \quad \boldsymbol{a}_t^T \boldsymbol{a}_t = \mathcal{P}_{tot}, \quad \boldsymbol{0} \preceq \boldsymbol{a}_t \preceq \sqrt{\boldsymbol{\mathcal{P}}_0}$$

$$\min_{\boldsymbol{\mathcal{P}}_t} . \quad |\boldsymbol{\mathcal{P}}_t - \boldsymbol{\mathcal{P}}_{t1}^*|^2 \quad (\mathcal{O}_4'') \qquad \text{s.t.} \quad \boldsymbol{1}^T \boldsymbol{\mathcal{P}}_t = \mathcal{P}_{tot}, \quad \boldsymbol{0} \preceq \boldsymbol{\mathcal{P}}_t \preceq \boldsymbol{\mathcal{P}}_0$$

- These sub-optimal solutions are good solutions when
  - $\kappa_1 = \frac{\mathcal{P}_{tot} \boldsymbol{g}^T \boldsymbol{g}}{c} \ll 1$ for $(\mathcal{O}_3)$,
  - $\kappa_2 = \frac{\mathcal{P}_{tot} \theta_{max}}{\sigma_n^2} \ll 1$, where $\theta_{max} = \max\{\theta_1, ...\theta_K\}$, for $(\mathcal{O}_4)$.

Maximizing MDC under IPC

$$\begin{array}{ll}\max\limits_{\mathbf{a}_t}. & \text{MDC}(\mathbf{a}_t) = \frac{\mathbf{a}_t^T \mathbf{b}_t \mathbf{b}_t^T \mathbf{a}_t}{\mathbf{a}_t^T \mathbf{K}_t \mathbf{a}_t + c} \quad (\mathcal{O}_5) \\ \text{s.t.} & \mathbf{0} \preceq \mathbf{a}_t \preceq \sqrt{\boldsymbol{\mathcal{P}}_0}\end{array}, \quad \begin{array}{ll}\max\limits_{\boldsymbol{\mathcal{P}}_t}. & \text{MDC}(\boldsymbol{\mathcal{P}}_t) = \frac{\boldsymbol{\mathcal{P}}_t^T \mathbf{b}_t \mathbf{b}_t^T \boldsymbol{\mathcal{P}}_t}{\boldsymbol{\mathcal{P}}_t^T \mathbf{K}_t \boldsymbol{\mathcal{P}}_t + \boldsymbol{\mathcal{P}}_t^T \mathbf{d}_t + c} \quad (\mathcal{O}_6) \\ \text{s.t.} & \mathbf{0} \preceq \boldsymbol{\mathcal{P}}_t \preceq \boldsymbol{\mathcal{P}}_0\end{array}$$

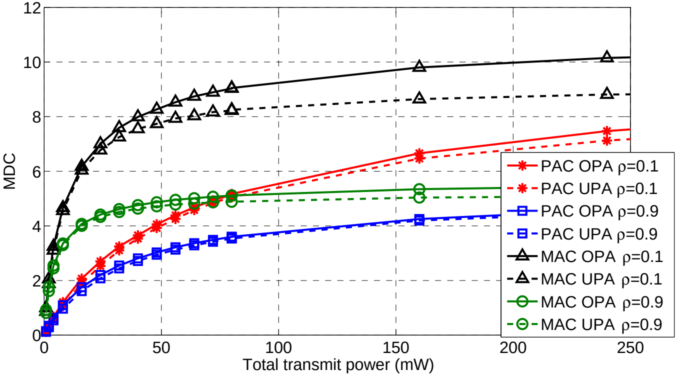
- $(\mathcal{O}_5)$: When $\kappa_3 = \frac{\mathbf{1}^T \boldsymbol{\mathcal{P}}_0 \mathbf{g}^T \mathbf{g}}{c} \ll 1$, the solutions are approximately $\mathbf{a}_t = \sqrt{\boldsymbol{\mathcal{P}}_0}$.
- $(\mathcal{O}_6)$: When $\kappa_4 = \frac{\mathbf{1}^T \boldsymbol{\mathcal{P}}_0 \theta_{max}}{\sigma_n^2} \ll 1$, where $\theta_{max} = \max\{\theta_1, ...\theta_K\}$, the solutions are approximately $\boldsymbol{\mathcal{P}}_t = \boldsymbol{\mathcal{P}}_0$.

Numerical Results-Setup

- $\mathcal{P}_{0_1} = \ldots = \mathcal{P}_{0_M} = \bar{\mathcal{P}}$, $\rho = 0.1, 0.9$, $M = 8$, $\epsilon_s = \epsilon_c = 2$, $\sigma_s^2 = 5\,\text{dBm}$, $\sigma_n^2 = -70\,\text{dBm}$, and $G = -55\,\text{dB}$.
- Sensors are deployed at on the circumference of a circle where its diameter is 5m. The source and FC coordinates are $(0, 0, 3)$ and $(0, 0, -10)$, respectively.
- $\mathcal{H}_0 : \; x_k = z_k, \; \mathcal{H}_1 : \; x_k = s_k + z_k, \; k = 1, \ldots, M$. $z_k \sim \mathcal{N}(0, \sigma_0^2)$.$s_k \sim \mathcal{N}(0, \sigma_{sk}^2)$ with $\sigma_{sk}^2 = \frac{\sigma_s^2}{d_{PS_k}^{\epsilon_S}}$.
- If $\boldsymbol{s} = [s_1, s_2, \ldots, s_M]^T$, then $K_s = \mathbb{E}\{\boldsymbol{s}\boldsymbol{s}^T\}$ where $K_s(i,j) = \rho_{ij}\sqrt{\sigma_{s_i}^2 \sigma_{s_j}^2}, \; \rho_{ij} = \rho^{d_{ij}}$. We assume ρ be the correlation at unit distance and d_{ij} is the distance between the sensors.
- We consider an energy detector at each sensor and maximize p_{d_k} at each sensor under the constraint $p_{f_k} < 0.1$. This results in $p_{d_k} = 0.6615$ for all the sensors.

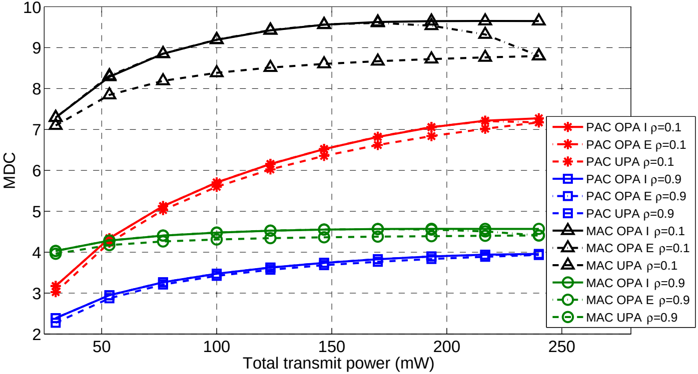
Numerical Results-TPC

- Low $\mathcal{P}_{tot}$: MAC outperforms PAC, High $\mathcal{P}_{tot}$: PAC converges MAC.
- Low $\mathcal{P}_{tot}$: the OPA and UPA have very close performance, High $\mathcal{P}_{tot}$: the gap between them is noticeable.
- As ρ increases, the difference between OPA and UPA decreases.



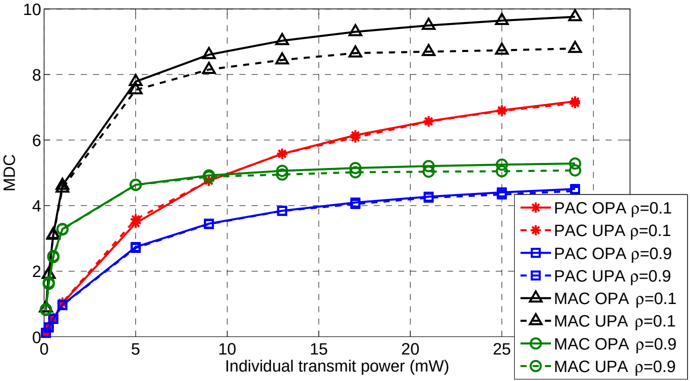
Numerical Results-TIPC

- Low $\mathcal{P}_{tot}$: E and I have the same performance, High $\mathcal{P}_{tot}$: there is a gap between them.



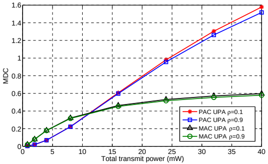
Numerical Results-IPC

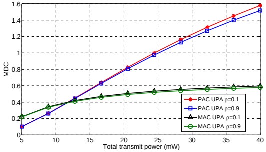
- Low $\bar{\mathcal{P}}$: performance of UPA and OPA are very close to each other.

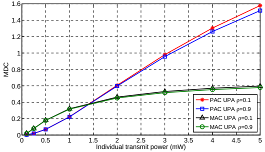


Numerical Results-Noncoherent

- High $\mathcal{P}_{tot}$ or $\bar{\mathcal{P}}$: PAC outperforms MAC, Low $\mathcal{P}_{tot}$: MAC performs better. By the increase of $\mathcal{P}_{tot}$ or $\bar{\mathcal{P}}$, correlation impact the MDC more noticeably.
- OPA and UPA have the same performance.

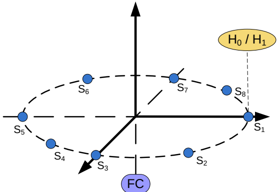






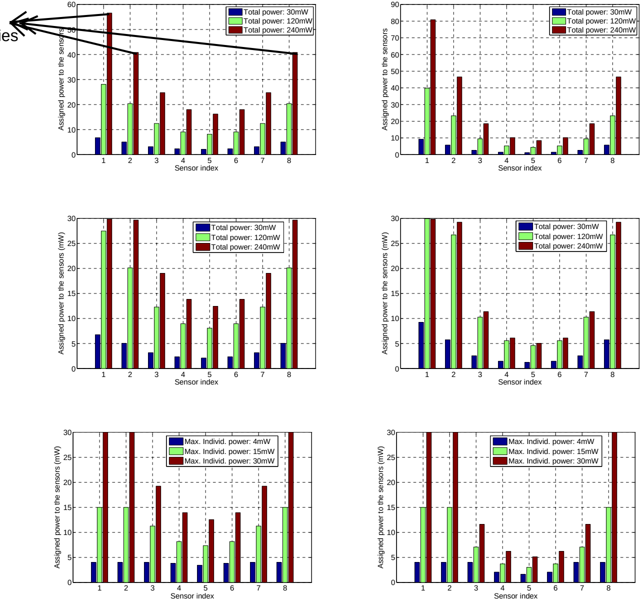
Numerical Results-Effect of Detection Indices on the Power Allocation

- We move the source to the point $(2.5, 0, 3)$.
- Optimal Power Allocations:
  - Under TPC or TIPC: sensors with higher p_{d_k} are assigned higher $\mathcal{P}_{t_k}$ for all the $\mathcal{P}_{tot}$ values.
  - Under IPC:
    - Low $\bar{\mathcal{P}}$: UPA is optimal,
    - High $\bar{\mathcal{P}}$: more power is allocated to the sensors with larger p_{d_k}.
- OPA in PAC is more similar to UPA than in MAC due to the lower noise variance in MAC.
- Higher ρ: more power is allocated to the sensors with larger p_{d_k}.

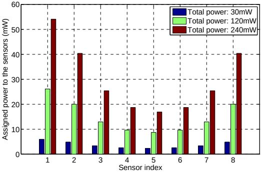


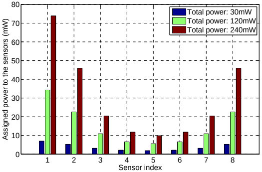
MAC Scheme and Coherent Reception

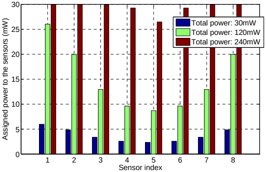
Highest probabilities of detection

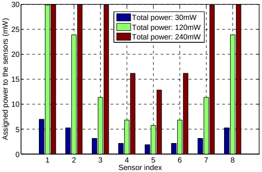


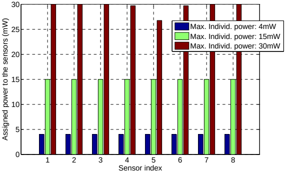
PAC Scheme and Coherent Reception

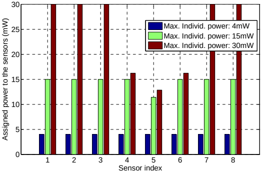






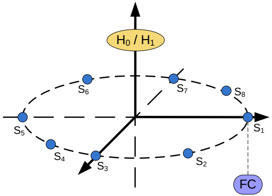




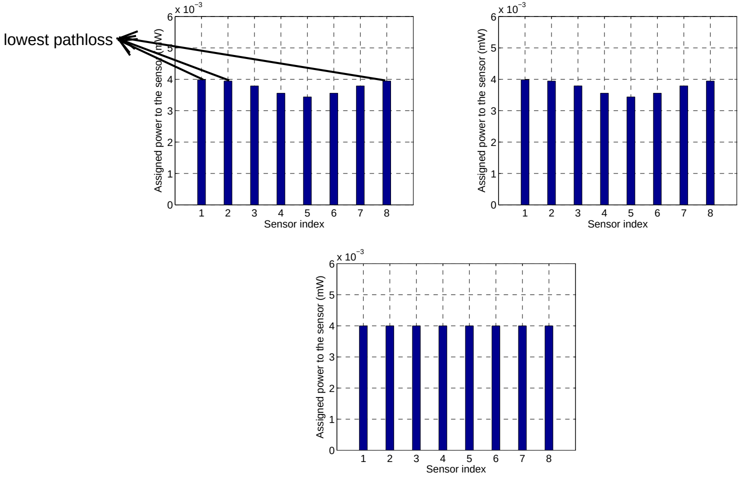


Numerical Results-Effect of pathloss between sensors and the FC

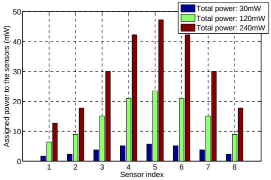
- We consider two scenarios:
  - Lower received power: FC is at $(2.5, 0, -10)$.
  - Higher received power: FC is at $(2.5, 0, -3)$.
- Optimal power allocation techniques:
  - Under TPC and TIPC
    - Lower $\mathcal{P}_{tot}$: water filling is the optimal power allocation technique,
    - Higher $\mathcal{P}_{tot}$: inverse water filling is the optimal power allocation.
  - Under IPC
    - Lower $\bar{\mathcal{P}}$: UPA is optimal,
    - Higher $\bar{\mathcal{P}}$: inverse water filling is optimal power allocation.

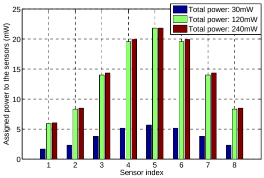


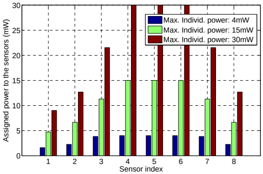
MAC Scheme Noncoherent Reception-Lower received power



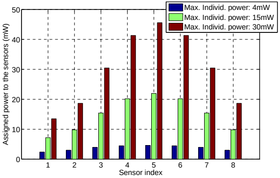
MAC Scheme Coherent Reception-Higher received power

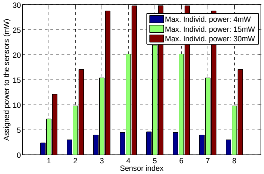


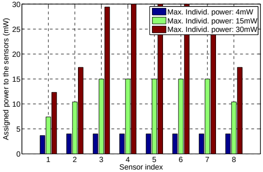




PAC Scheme Coherent Reception-Higher received power







Conclusion

- P1: We have proposed three new architectures. There is no explicit information exchange in scheme (iii).
  - Our numerical results show that, unless for low communication SNR and moderate/high sensing SNR, performance improvement is feasible with the new cooperative and parallel fusion architectures, while scheme (iii) outperforms others.

Conclusion

- P1: We have proposed three new architectures. There is no explicit information exchange in scheme (iii).
  - Our numerical results show that, unless for low communication SNR and moderate/high sensing SNR, performance improvement is feasible with the new cooperative and parallel fusion architectures, while scheme (iii) outperforms others.
- P2: We considered linear fusion rule with spatially correlated observations, coherent and noncoherent PAC and MAC schemes. We designed optimal linear fusion rule with PAC scheme and found optimal sensor power allocation for PAC and MAC under TPC and IPC on sensors' power.