

Fuzzy Modeling of Drivers' Actions at Intersections

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Abstract—Advanced Driver Assistance systems (ADAS) are systems that assist the driver during the driving task. This technology has great potentials in improving driver and traffic safety. It is very important for an ADAS to predict human drivers' behaviors at urban environment to avoid crashes. Because of the complexity of human-vehicle interaction, it is difficult to obtain an explicit model for analyzing the drivers' behaviors. Instead, models are developed for various driver decisions and driving scenarios (such as lane change decisions and intersection scenarios) which can then be integrated using switch models. Intersections are one of the major scenarios that require special attention in driver behavior modeling. This paper uses Takagi-Sugeno as a data driven technique to model and predict drivers' behaviors at intersections. In the proposed technique, a Takagi-Sugeno model is trained for each maneuver using a Gath-Geva fuzzy clustering algorithm. The proposed models are then evaluated with naturalistic real-world driving data collected in urban traffic, and the estimation results are presented. The results suggest that the proposed technique can correctly estimate the drivers' actions at intersections with high accuracy. This technique uses fewer numbers of maneuver models for training that leads to less computational complexity.

I. INTRODUCTION

According to US Department of Transportation (DOT) traffic accidents are the cause for around 37,000 fatalities and 230.6 billion dollars economic loss every year in US [1]. Most of these accidents are due to reckless driving (speeding, distractions, etc.). These statistics magnify the need for an ADAS that is able to take over partial or full control of the vehicle when necessary in order to avoid possible hazards. The distribution of pedestrians and other infrastructure in urban areas pose a major challenge to the development of ADAS. The higher number of humans and vehicles makes recognition of a possibly dangerous situation more difficult. Vehicles tend to move towards full autonomy in attempts to develop ADASs for highway/freeway environments due to very limited interactions in these environments relative to their urban counterparts [2].

An ADAS in urban environment is required to generate a situational awareness of the surroundings which involves modeling human driver activities in order to make meaningful decisions. Hence, it can predict other drivers' actions and avoid a possible crash. Within the urban environments, intersection scenarios are very challenging in the development of ADAS because several parameters including traffic light condition, time of day, etc. are involved in modeling the human driver

behavior in those situations. Moreover, people drive differently based on their age, gender and environment. Thus, for a comprehensive model of a human driver, a variety of drivers' behaviors need to be collected and analyzed. Machine learning techniques are very useful in handling and analyzing such stochastic behaviors. These features make them very popular in the literature related to modeling and prediction of human drivers.

Modeling driver behavior at signalized intersections when signal is yellow has been of significant interest over the past couple of years [3]. Driver's Perception Reaction Time (PRT) at the onset of a yellow traffic light was found to be dependent on driver's age and gender as well as the driving condition (following a car or being followed) [4]. A fuzzy logic model was developed in [5] based on the vehicle's speed and distance to the intersection. Factors affecting driver's action during a yellow light were investigated by developing a nested logit model applied to a set of naturalistic driving data [6]. It was reported that a shorter duration of yellow light leads to red light running (RLR). These models can improve the understanding of driver behavior at signalized intersections and thus increase the traffic safety. In addition, "driver aggressiveness" is introduced in [7] as a new variable for modeling the behavior of drivers. Three well-known machine learning techniques: k nearest neighbor, random forest and adaptive boosting were used on a set of experimental data and the results indicated that adding the new predictor has increased the model accuracy for all the methods. However, the "driver aggressiveness" variable should be obtained from history of the driver, and is only usable through V2V communications. Thus, it cannot be used as a common measure such as velocity.

Probabilistic approaches have also been used by some researchers to model driver behavior in order to overcome the challenges caused by the uncertainty of the data. For instance, a Bayesian model was developed for modeling and prediction of the driving task [8]. A dynamic Bayesian network was used to make lane changing decisions, after the situation is assessed by the image processing algorithm [9]. Some other researchers have tried to predict driver behavior using clustering or classification algorithms. Future behavior of the driver was also predicted by trajectory classification [10]. Agglomerative Complete-Link Hierarchical Clustering (CL) and Deterministic Annealing Pair-wise Clustering (DA) are used in [11] to estimate vehicle motion. However, the inability

of these techniques to capture the variable dynamic of an intersection situation make them weak in modeling a driver's behavior at an intersection.

Some approaches have specifically focused on modeling drivers' behavior at intersections [12]. One approach involved the rule based estimation in which the relationship between vehicle dynamics and driver states were predefined through a set of rules [13]. The disadvantage of this technique is that for a complex model it is difficult and inefficient to define all the possible states and assign their transition probabilities. Another technique for modeling the driver behavior is Hidden Markov Model (HMM) estimation which describes a dynamic process through two random processes. This method is popular because of its capability of modeling the variable dynamics, such as other vehicles and pedestrians, at the intersection; also, it is more versatile and requires less external input [14]. In [15], a model for estimation of driver behavior at intersections is proposed that uses a combination of multi-class Support Vector Machines (SVM) and Hybrid State Systems (HSS). This technique employs statistical feature extraction to be able to represent the historical information of the data. The simulation results suggest that it has a better performance than the combination of HMMs and HSS.

In this paper, the order of the data in time is considered for modeling the driver's behavior at intersections with a nonlinear second order difference equation. Then, due to the uncertainties in an individual's actions, the nonlinear function is approximated by a Takagi-Sugeno (TS) fuzzy model. A modified fuzzy clustering algorithm is employed for this approximation. In the second section, the proposed modeling approach is introduced. Also, the Gath-Geva clustering algorithm for identifying parameters of TS model, as well as a proposed two step evaluation procedure are briefly explained. The third and fourth sections include data collection, simulations and results respectively. Finally, conclusions and future work are given in the last section.

II. PROPOSED METHOD FOR HUMAN DRIVER MODELING

In this section, the proposed modeling technique for driver behavior is presented. In the first part, the nonlinear second-order difference function model for driver behavior is presented. Then, the TS local models approximation using the Gath-Geva clustering algorithm is explained.

A. Definition of Driver Behavior Nonlinear Models

In this study, when a driver arrives at an intersection, there are four possible maneuvers: go straight (GS), turn right (TR), turn left (TL), or stop (S). In order for the ADAS to predict a vehicle's motion, it requires some information such as velocity, distance, etc. about the vehicle. This data can be obtained through GPS, sensors, Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) communications.

For our previous study in modeling driver behavior using fuzzy clustering [16]; velocity, acceleration and yaw-rate were considered as the measurement inputs for the proposed method. Also, two models were trained for each observation of

a maneuver according to the velocity; a fast model and a slow model. In this study, the same three observation inputs are used to model two main variables: velocity and yaw rate, which are the main characteristics required for modeling a vehicle's motion on a 2D plane according to the bicycle model [17]. For each maneuver it is assumed that velocity (V) and yaw-rate (ω) are nonlinear functions of their corresponding inputs at one or two time steps before. The two time steps before is taken into consideration for more accuracy. In other words, the velocity model is assumed to be a function of velocity and acceleration at one step before, and yaw-rate model to be a function of yaw-rate at one and two time steps before. As an example, the TL maneuver is given in eq. (1):

$$\begin{aligned} V_{left}(t+1) &= f_1(V_{left}(t), A_{left}(t)) \\ \omega_{left}(t+1) &= f_2(\omega_{left}(t), \omega_{left}(t-1)) \end{aligned} \quad (1)$$

It is very difficult to identify an explicit global model for such systems using basic physical principles. Since the TS fuzzy technique is a universal approximator and is able to approximate any nonlinear function, it is employed for modeling the driving system [18], [19]. The idea behind Takagi-Sugeno models is that instead of estimation of a complex model with one linear function, smaller local models are determined over various regions that are fuzzily defined [20].

This model consists of a set of "if-then" rules with fuzzy membership functions in the antecedent and a linear function in the consequent part of the rule, as in eq. (2).

$$R_i = \text{if } x \text{ is } A_i(x), \text{ then } \hat{a} = a_i^T x + b_i. \quad i = 1, \dots, c \quad (2)$$

Where $A_i(x)$ is a membership function that determines the firing degree of each rule, a_i and b_i are parameters of the local linear model, and c is the number of rules. In other words, the consequent functions are the local models in the regions specified by the antecedent membership functions [21], [22]. For example, if the firing degrees of $V(t)$ and $A(t)$ are a_1 and a_2 and the constant is b , then $V(t+1) = a_1V(t) + a_2A(t) + b$.

B. Estimation of Driver Nonlinear Models using Modified Gath-Geva Fuzzy Clustering

In this section, the unknown parameters of the estimated yaw-rate and acceleration models are calculated for each of the maneuvers. The first step is to determine the fuzzy membership functions in the antecedent ($A_i(x)$ in eq. (2)). The membership functions are then used to calculate the coefficients of the local linear model in the consequent (a_i, b_i) in eq. (2). Several techniques exist in the literature for finding the linear function's parameters, such as genetic algorithm, least-square, clustering and model based approaches [23]. Here, the modified Gath-Geva fuzzy clustering algorithm introduced in [24] is employed for identification of the TS models. The modified GG algorithm uses a Gaussian mixture model for the membership functions and calculates its parameters from the data. Then, Least Square estimation is applied to obtain the

coefficients of the consequent. This method integrates Gath-Geva Fuzzy clustering with expectation maximization (EM) of the Gaussian mixture models. The algorithm was developed in [24] and the procedure for identification of the TS models is as follows:

These steps are repeated for $l = 1, 2, \dots$ iterations.

Step 1: Calculate cluster parameters

Centers of the membership functions ($v_{(i,j)}$), Standard deviation of the membership functions ($\sigma_{i,j}^2$), and Parameters of the local models (θ_i) using least square (LS) error estimation as follows:

$$v_i = \frac{\sum_{k=1}^N \mu_{i,k}^{l-1} x_a}{\sum_{k=1}^N \mu_{i,k}^{l-1}} \quad (3)$$

$$\sigma_{i,j}^2 = \frac{\sum_{k=1}^N \mu_{i,k}^{l-1} (x_{j,k} - v_{j,k})^2}{\sum_{k=1}^N \mu_{i,k}^{l-1}} \quad (4)$$

$$\theta_i = (X_e^T \phi X_e)^{-1} X_e^T \phi X_e y \quad (5)$$

Where x_k is the input data that we want to cluster, y is the output data from the training set, $\mu_{i,k}$ is its membership degree, $X_e = [X \ 1]$ is the regressor matrix and ϕ is a matrix with the membership degrees on its main diagonal.

Step 2: Compute the distance measures

The distance measure is a combination of the distance between the cluster center and data points x , and the performance measure of the obtained local TS models.

$$\frac{1}{D_{i,k}} = \prod_{j=1}^n \frac{\alpha_i}{\sqrt{2\pi\sigma_{i,j}^2}} \exp\left(-\frac{1}{2} \frac{(x_{j,k} - v_{j,k})^2}{\sigma_{i,j}^2}\right) \cdot \frac{1}{\sqrt{2\pi\sigma_{i,j}^2}} \cdot \exp\left(-\frac{(y_k - f_i(x_k, \theta_k))^T (y_k - f_i(x_k, \theta_k))^T}{2\sigma_i^2}\right) \quad (6)$$

Step 3: Update the partition matrix

$$\mu_{i,k}^{(l)} = \frac{1}{\sum_{j=1}^c \left(\frac{D_{i,k}(z_k, \eta_i)}{D_{j,k}(z_k, \eta_j)}\right)^2} \quad (7)$$

One advantage of this technique is that it does not use transformed input variables; therefore, the results are more easily interpretable. Another advantage is that the number of rules is equal to the given number of clusters, which reduces the computation complexity of a large number of rules.

C. Two Step Evaluation Procedure

It was mentioned in the previous sections that a model for each maneuver is trained with observations from that maneuver. This means that a correctly estimated maneuver should result in the smallest error between the actual and estimated measurements. However, due to the experimental nature of the data as well as the presence of noise, both the

velocity and yaw-rate models may not lead to the smallest error value simultaneously.

A voting strategy was utilized in relevant studies [16]; in which three models were developed for each maneuver. However, voting is impractical in this study as there are only two models for each maneuver. Therefore, a new priority based two-step evaluation procedure is propose.

When trying to predict the driver’s action at the intersection, driver decision can be categorized into two major groups - whether the driver stops at or crosses the intersection. If the driver decides to stop, then the environment is clear for the vehicles in the opposing lanes to continue their path. But if the estimation indicates that driver is to cross the intersection, the actions of other vehicles need to be adjusted to avoid any crashes.

In the proposed evaluation procedure, first stopping or passing the intersection is investigated. Among the three measurements available, velocity can model the stop-or-go action at intersections better than the other measurements. Therefore, the measured velocity of the event is compared with the estimated velocities from all the four maneuver models. If the “stop” model matches the measured velocity best, it is considered as the intended action at the intersection and the evaluation ends.

If the measured velocity does not match the “stop” model, the three remaining maneuvers “straight”, “right turn” and “left turn” are compared with each other. Velocity profile of these maneuvers is largely dependent on the situation and mostly similar, as the vehicles usually decelerates specially for right and left turns. However, in some scenarios such as yellow traffic light some drivers may accelerate to pass the intersection quickly. Therefore, while velocity is a good indication of motion or no motion at intersection, it is not a reliable indication of the direction of the motion.

The main difference between the three mentioned maneuvers is yaw-rate, since it is the only available measurement that can describe the lateral movement. Yaw-rate is positive for “right turn”, negative for “left turn” and oscillates around zero for “straight” maneuvers. Therefore it is a suitable measure to differentiate the various turns from straight maneuver. In the second step of the proposed evaluation, yaw-rate models of each maneuvers use the yaw-rate measurements to estimate future values. The model that has the smallest error is chosen as the intended action of the driver.

III. DESCRIPTION OF EXPERIMENTAL DATA

In order to evaluate the proposed method, the mathematical models explained previously need to be applied to driving data. In this paper a set of naturalistic driving data collected by the Ohio State University (OSU) is used.

The vehicle used for the data collection experiment was a 2012 Honda Accord equipped with the following sensors: NovAtel GPS unit- it provides GPS latitude, GPS longitude, times tamp of reading and others; controller area network (CAN) bus- it provides timestamp of reading, yaw rate, lateral acceleration, speed, and others. The data is recorded by the

sensors at a rate of 0.1 seconds. Three HD cameras- they provide views of the front, left side, and right side of the vehicle. Since the objective is to estimate the motion of the vehicle based on its dynamic characteristics, it is assumed that these sensors measure all the information required for the models.

For this experiment, participants were recruited to drive the sensor fitted car around the streets of Columbus, OH (Fig. 1). The paths were carefully selected to represent everyday routes.



Fig. 1: Videos from the OSU sensor fitted vehicle used for data collection

The data extraction process was performed manually by marking the collected videos. The center of the intersection was considered as the origin. The start and end of the time span were considered to be 10 seconds before and 10 seconds after crossing the intersection respectively. For example, when the vehicle turns left at an intersection in the video, that time span is marked and the corresponding data (velocity, acceleration and yaw-rate) time series were extracted.

IV. MODEL TRAINING AND SIMULATIONS

As mentioned in the previous section, in the proposed modeling technique a nonlinear relationship is considered between the values of velocity, acceleration and yaw-rate of a maneuver with its observations at one or two-steps before. Then, these nonlinear systems are approximated with the TS local models using a Gath-Geva fuzzy clustering based algorithm. Each local model has two inputs. The two inputs for the velocity model are velocity and acceleration at one step before, and the two inputs for the yaw-rate model are yaw-rate values at one and two time steps before. The training data for each model input consists of two time series from two events that are concatenated into one large time series. An example of the membership functions of velocity model for “right turn” maneuver is shown in Fig 2.

In this method, there are four maneuvers (straight, stop, left, right) and two observation models (velocity, yaw-rate) for each. So in total eight models are trained. Once all the TS models are identified for the four maneuvers, the model is tested. A specific incident is considered as testing data and its velocity and yaw rate are fitted to the trained models. At last, the two step evaluation procedure is performed. First, velocity and acceleration are used as inputs to velocity models of all the

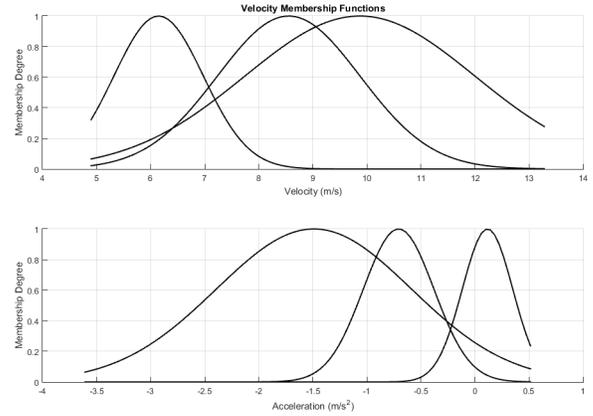


Fig. 2: Membership functions for the velocity TS model of Right Turn maneuver

TABLE I: total number of maneuvers in the experimental data set

Maneuver	Occurrence
Straight	5
Stop	5
Right turn	4
Left Turn	5

four maneuvers. If the “stop” model estimated velocity results in the best match to the actual velocity of the vehicle, “stop” will be considered as the intended action at the intersection. Otherwise, the yaw-rate models of the remaining maneuvers determine the estimated action. The maneuver that has its yaw-rate output being the most similar to the measured yaw-rate will be chosen as the estimated action at the intersection.

The test data includes samples of observations from different maneuvers (labeled data). The observation values of the testing data are each fitted to their respective models of each maneuver and Mean Squared Error (MSE) is calculated as the performance measure for each model.

Table I shows the number of samples for each maneuver in the testing data. Tables II and III show the results obtained when samples of different maneuvers is tested. In each of these tables, the combination of MSEs from all the models of maneuvers is presented as a matrix. The rows of the matrix represent the maneuvers: straight, right turn, left turn and stop, respectively. The columns of the matrix represent the MSE values of velocity and yaw rate for each maneuver respectively.

In table II, the velocity measurements of a stop maneuver is tested by all the four velocity models. It is obvious that the second row of the MSE matrix that corresponds to the “stop” model has the smallest error. Fig. 3 shows the estimated

TABLE II: Velocity MSE table for example 1 (stop maneuver)

Maneuver	Estimation Error
Straight	0.0256
Stop	0.0034
Right turn	0.0160
Left Turn	0.0117

TABLE III: Velocity and yaw-rate MSE table for example 2 (right turn maneuver)

Maneuver	Velocity Model	Yaw-rate Model
Straight	0.0924	165.3580
Stop	13.3340	166.0013
Right Turn	0.0541	0.1776
Left Turn	1.4364	124.8819

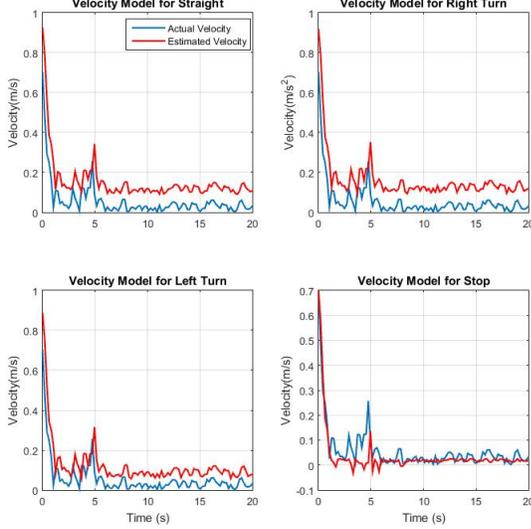


Fig. 3: Estimated output from the velocity model of the four maneuvers based on stop input data

outputs from different velocity models with the “stop” turn data as the input. It can be seen that the stop model has the best estimation performance and therefore its corresponding error is very small, which is consistent with table II.

In the second example, right turn data was tested. The velocity model test shows that the test data does not match the “stop” model, which means the driver is not planning to stop at the intersection. It has to be noted that although in this example the “right turn” velocity has the smallest MSE, it may not always be the case. A small MSE value for any model other than the “stop” will not give an accurate estimation of the driver action. Therefore, the yaw-rate models are utilized to determine the driver’s action in crossing the intersection. The MSE column for the yaw-rate models demonstrates that the “right turn” model results in the smallest error, which implies that it is the best fit for the test time-series.

The estimated yaw-rate using the right turn data by the three remaining maneuver models is shown in Fig. 4. It is depicted that the straight and stop models that are trained with very small yaw-rate values are not able to estimate the right turn yaw-rate signal. The left turn model has a slightly better estimation performance than stop and straight at the beginning, but it fails to estimate as the values get larger. On the other hand, the “right turn” model has a good estimation performance the entire time.

Finally, the confusion matrix is given in Table IV. This

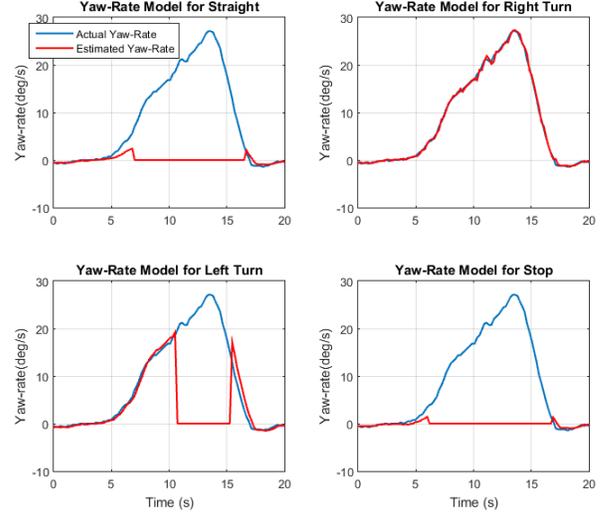


Fig. 4: Estimated output from the yaw-rate model of the four maneuvers based on right turn input data

TABLE IV: Confusion matrix for the proposed method

	Straight	Stop	Right	Left
Straight	4	0	1	0
Stop	0	5	0	0
Right	0	0	4	0
Left	0	0	1	4

matrix is used to show the classification performance of the proposed technique. It should be noted that each of the observations of an instance, are a time-series consisting of 200 points, collected during 20 seconds. The rows of this matrix represent the actual maneuver, and the columns represent the estimated maneuver. Thus, it is easy to check how many instances are correctly estimated out of the whole dataset.

Misclassification may be attributed to the effect of measurement noise on the model training. Also, in case of the stop and straight, there are some signs of right or left turn in the yaw-rate signal from the previous or next maneuvers. These differences in the yaw-rate lead to poor modeling and higher MSE; consequently, the estimation will be wrong.

V. CONCLUSION

In this paper, a fuzzy based modeling technique is introduced for estimation of driver behavior at intersections. The required observations for modeling are velocity, acceleration and yaw-rate which are selected according to vehicle motion equations. In the proposed method, velocity and yaw rate are nonlinear functions of their values at one and two time steps before. These nonlinear functions are approximated with local Takagi-Sugeno models using a Gath-Geva fuzzy clustering technique. The model is trained and then tested with naturalistic driving data from the OSU. The simulation results show that the model has a good estimation performance.

One of the contribution of this fuzzy technique is its consideration of the order of the data in time-series, as it

employs a second order difference equation for the maneuver contrary to other classification techniques. The SVM and k nearest neighbor only consider data point by point without any order. In contrast, this work differs from previous study [16] in the sense that separate models are not trained according to fast or slow velocity, stop maneuver is added to the algorithm. These modifications have resulted in more accuracy and less computationally expensive estimations. The next contribution in this study is the two step evaluation process. First it eliminates the stopping intent, and later determines the trajectory (straight, right or left). The implementation of this process reduces the computational complexity by reducing the unnecessary estimation of the driver's direction if he/she is stopping first. Consequently, the reaction time can be reduced in situations that everyone follow traffic regulations.

Future work will involve the expansion of this method to prediction of driver's future actions rather than estimation of the next move. Integration of V2V and V2I communications into the study will also be explored in order to increase knowledge base relating to other vehicles such as the traffic signal of the opposing lane, driving behavior of the other drivers, their intended paths, etc. This will help in improving the accuracy of prediction.

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REFERENCES

- [1] A. for Safe International Road Travel. Road crash statistics. [Online]. Available: <http://asirt.org/Initiatives/Informing-Road-Users/Road-Safety-Facts/Road-Crash-Statistics>
- [2] T. Streubel and K. H. Hoffmann, "Prediction of driver intended path at intersections," in *Intelligent Vehicles Symposium Proceedings, 2014 IEEE*. IEEE, 2014, pp. 134–139.
- [3] T. Hicks, R. Tao, and E. Tabacek, "Observations of driver behavior in response to yellow at nine intersections in maryland," in *Transportation Research Board 84th Annual Meeting, Washington, DC*, 2005.
- [4] I. El-Shawarby, H. A. Rakha, A. M. M. Amer, and C. McGhee, "Characterization of driver perception reaction time at onset of yellow indication," in *Transportation Research Board 89th Annual Meeting*, no. 10-0520, 2010.
- [5] D. S. Hurwitz, H. Wang, M. A. Knodler, D. Ni, and D. Moore, "Fuzzy sets to describe driver behavior in the dilemma zone of high-speed signalized intersections," *Transportation research part F: traffic psychology and behaviour*, vol. 15, no. 2, pp. 132–143, 2012.
- [6] T. Gates, P. Savolainen, and H.-U. Maria, "Prediction of driver action at signalized intersections by using a nested logit model," *Transportation Research Record: Journal of the Transportation Research Board*, no. 2463, pp. 10–15, 2014.
- [7] M. Elhenawy, H. Rakha, and I. El-Shawarby, "Enhanced modeling of driver stop-or-run actions at a yellow indication: Use of historical behavior and machine learning methods," *Transportation Research Record: Journal of the Transportation Research Board*, no. 2423, pp. 24–34, 2014.
- [8] A. Gerdes, "Driving manoeuvre recognition," *Tagungsband zum ITS*, 2006.
- [9] R. Schubert, K. Schulze, and G. Wanielik, "Situation assessment for automatic lane-change maneuvers," *Intelligent Transportation Systems, IEEE Transactions on*, vol. 11, no. 3, pp. 607–616, 2010.
- [10] C. Hermes, C. Wohler, K. Schenk, and F. Kummert, "Long-term vehicle motion prediction," in *Intelligent Vehicles Symposium, 2009 IEEE*. IEEE, 2009, pp. 652–657.
- [11] F. Large, D. Vasquez, T. Fraichard, and C. Laugier, "Avoiding cars and pedestrians using velocity obstacles and motion prediction," in *Intelligent Vehicles Symposium, 2004 IEEE*. IEEE, 2004, pp. 375–379.
- [12] V. Gadepally, A. Kurt, A. Krishnamurthy, and U. Ozguner, "Driver/vehicle state estimation and detection," in *Intelligent Transportation Systems (ITSC), 2011 14th International IEEE Conference on*. IEEE, 2011, pp. 582–587.
- [13] A. Kurt, J. L. Yester, Y. Mochizuki *et al.*, "Hybrid-state driver/vehicle modelling, estimation and prediction," in *Intelligent Transportation Systems (ITSC), 2010 13th International IEEE Conference on*. IEEE, 2010, pp. 806–811.
- [14] V. Gadepally, A. Krishnamurthy, and U. Ozguner, "A framework for estimating driver decisions near intersections," *Intelligent Transportation Systems, IEEE Transactions on*, vol. 15, no. 2, pp. 637–646, 2014.
- [15] S. B. Amsalu, A. Homaifar, F. Afghah, S. Ramyar, and A. Kurt, "Driver behavior modeling near intersections using support vector machines based on statistical feature extraction," in *Intelligent Vehicles Symposium (IV), 2015 IEEE*. IEEE, 2015, pp. 1270–1275.
- [16] S. Ramyar, M. G. Sefidmazgi, S. Amsalu, A. Anzagira, A. Homaifar, A. Karimodini, and A. Kurt, "Modeling driver behavior at intersections with takagi-sugeno fuzzy models," in *Intelligent Transportation Systems (ITSC), 2015 IEEE 18th International Conference on*. IEEE, 2015, pp. 2378–2383.
- [17] R. Vasudevan, V. Shia, Y. Gao, R. Cervera-Navarro, R. Bajcsy, and F. Borrelli, "Safe semi-autonomous control with enhanced driver modeling," in *American Control Conference (ACC), 2012*. IEEE, 2012, pp. 2896–2903.
- [18] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *Systems, Man and Cybernetics, IEEE Transactions on*, no. 1, pp. 116–132, 1985.
- [19] A. Homaifar, M. Bikdash, and C. Clifton, "Approximating an optimal feedback control law by a generalized sugeno controller," *Fuzzy sets and systems*, vol. 121, no. 1, pp. 39–57, 2001.
- [20] P. P. Angelov and D. P. Filev, "An approach to online identification of takagi-sugeno fuzzy models," *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on*, vol. 34, no. 1, pp. 484–498, 2004.
- [21] M. Sugeno and G. Kang, "Fuzzy modelling and control of multilayer incinerator," *Fuzzy sets and systems*, vol. 18, no. 3, pp. 329–345, 1986.
- [22] C. Clifton, A. Homaifar, and M. Bikdash, "Design of generalized sugeno controllers by approximating hybrid fuzzy-pid controllers," in *Fuzzy Systems, 1996., Proceedings of the Fifth IEEE International Conference on*, vol. 3. IEEE, 1996, pp. 1906–1911.
- [23] A. Homaifar and E. McCormick, "Simultaneous design of membership functions and rule sets for fuzzy controllers using genetic algorithms," *Fuzzy Systems, IEEE Transactions on*, vol. 3, no. 2, pp. 129–139, 1995.
- [24] J. Abonyi, R. Babuška, and F. Szeifert, "Modified gath-geva fuzzy clustering for identification of takagi-sugeno fuzzy models," *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on*, vol. 32, no. 5, pp. 612–621, 2002.