Design of $K$—user Massive MIMO Networks

Anil Kumar Yerrapragada*, Brian Kelley†

Department of Electrical and Computer Engineering, University of Texas at San Antonio, San Antonio, Texas 78249
Email: *anilkumar.yerrapragada@utsa.edu, †dr.brian.kelley@gmail.com

Abstract—Network densification enables ultra high capacity gains in 5G networks by shrinking cell sizes, thereby bringing users closer to base stations. However, this leads to severe levels of interference. A promising method to suppress interference in $K$—user MIMO networks is interference alignment. Massive MIMO is another technology that is expected to contribute to increased capacity by the deployment of hundreds of transmit and receive antenna elements. In this paper we describe a design methodology for evolved 5G dense networks using interference alignment and $K$—user MIMO techniques. Our contributions include mathematical models showing the extension of the interference alignment protocol for $K > 3$ users using a modified hypercube network structure. We also provide a model for interference alignment under Cramér-Rao bound channel estimation error constraints. Further, we show how large numbers of transmit and receive antennas can provide additional signal power gains and present a user scheduling protocol to maximize capacity.

Keywords—$K$-user MIMO, Dense Networks, Massive MIMO

I. INTRODUCTION

As per [1] there has been a near exponential growth in our demands for data and in the number of connected devices. To support this, evolved 5G networks need to be able to deliver a 1000x increase in capacity and 10-100x higher number of connected devices [2], [3]. The authors in [4] ran a survey which revealed that increased network throughput was the main expectation from 5G networks. A key enabler for this increase is network densification.

A. Network Densification—Merits and Challenges

Network densification involves configuring networks with micro and femtocells each with its own Remote Radio Head (RRH). This brings users closer to the network and allows for higher receiver signal powers and capacity. The major challenge of network densification is the unprecedented interference that results from bringing RRHs and users closer together. The works presented in [5] and [6] detail the different types of interference scenarios that effect ultra dense networks. It is clear that to fully reap the benefits that network densification promises, it is necessary for networks to incorporate some form of interference suppression protocol.

B. Prior Research on $K$—User MIMO and Interference Alignment

A $K$—user network is one in which there are $K$ transmitters and $K$ receivers. Each receiver receives signals from $K$ transmitters in a Co-ordinated Multipoint (CoMP) fashion.

Interference alignment achieves interference cancellation in $K$—user networks by aligning interfering signals in such a way that the interference free space is maximized. Precoders are applied to the transmit signals and beamformers to the receive signals. The number of interference-free dimensions is defined as Degrees of Freedom (DoF) [7], [8]. It has been long accepted that a maximum of only 1 DoF could be achieved in $K$ transmitter and $K$ receiver networks. There are several works in this area, of which [7] analyses the feasibility of $K^2$ DoF. Several other examples showing the application of interference alignment to $K$—user MIMO networks are provided in [9]. We have presented an interference alignment based self-organizing architecture for interference suppression in dense Internet of Things environments in our previous work [10]. Lastly, the authors of [11] have shown an interference alignment algorithm that achieves perfect interference suppression using zero forcing beamformers for a $K = 3$ system. This framework can achieve $K^2$ DoF across the $K$ users which is an improvement from the work in [7]. To the best of our knowledge, the main demerit of the prior research on interference alignment is that there is no mathematical framework that deals with the extension of $K$-user MIMO protocol to higher values of $K > 3$. Further, the integration of $K$-user MIMO and massive MIMO frameworks has not been considered in prior research.

C. Conversion of $K$—User MIMO to a Massive MIMO Framework

The chief characteristic of Massive MIMO is the very large number of antenna elements, of the order of hundreds or thousands, installed at the base stations. By operating these antennas in a coherent manner, ultra-high gains can be achieved. This is shown in [12] where closed form expressions for spectral efficiency are derived. From a resource allocation standpoint, the authors have addressed the question of how many pilots (for channel estimation) and users need to be scheduled in each time slot to maximize spectral efficiency. Other expressions for sum rate capacity, based on statistical beam forming and greedy search user scheduling is presented in [13]. In [14], application of massive MIMO to 5G dense networks is discussed. The authors have investigated the deployment of clustered small cells with 2 dimensional antenna arrays. User association algorithms like rate maximization and proportional fairness in heterogeneous networks employing massive MIMO are presented in [15].

978-1-5386-1104-3/17/$31.00 © 2017 IEEE
The difference between a $K$-user MIMO network and the massive MIMO networks such as those presented in most of the works above is the absence of a CoMP framework. Each receiver receives signals from only one base station. Further, only the base stations are assumed to be equipped with massive MIMO while users are assumed to have single antennas. In our $K$-user system, both transmitters and receivers are equipped with $M$ antennas where $M = K(K-1)$. From this, it is clear that as $K$ increases, $M$ scales quite rapidly into the realm of massive MIMO.

D. Our Contributions

In this paper, we present an interference alignment algorithm for $K=3$ user MIMO networks. We have found that the work in [11] cannot be easily extended to higher values of $K$. To address this, we have presented a modified $K$-user network model based on hypercube connections between nodes and provide realistic models for implementation under Cramér-Rao bound constraints. We also present an insight into the capacity gains that are possible due to the interference alignment.

Our Contributions

In our proposed protocol for design of 5G dense networks with interference alignment. Figure 1 shows the steps in our proposed protocol for design of 5G dense networks with interference alignment.

II. INTERFERENCE ALIGNMENT SCHEME

A. K User Network Model

The precoded signal vector $x_j$ of length $M$, from the $j^{th}$ transmitter to the $i^{th}$ receiver, is given by

$$x_j = \sum_{i=1}^{K} v_{ij} s_{ij}$$

(1)

The power of the transmit signal from each transmitter is subject to following constraint,

$$Tr(x_j^H x_j) \leq P_j$$

(2)

Where $P_j$ is the total power available at the $j^{th}$ transmitter.

Signals are transmitted over Rayleigh fading channels. We denote the full rank channel matrix from the $j^{th}$ transmitter to the $i^{th}$ receiver by $H_{ij}$. The noise is denoted by $n_i$ which we assume is Gaussian with zero mean.

Fig. 1: Flowgraph showing the steps in our proposed 5G dense network protocol

B. Zero Forcing Beamformer

At each receiver, a beamformer matrix is applied to the received signal. The function of the beamformer is to null the interference component as shown below. $\tilde{y}_i$ is the signal after beamforming and is given by

$$\tilde{y}_i = U_i^H \sum_{j=1}^{K} H_{ij} v_{ij} s_{ij} + U_i^H \sum_{j=1}^{K} \sum_{k=1}^{K} H_{ij} v_{kj} s_{kj} + U_i^H n_i = 0$$

(3)

Perfect interference cancellation can be achieved subject to the following constraints.

Postulate 1 [11]: At each receiver, interfering signals coming from the same transmitter, cannot be aligned in the same direction. This can be represented as follows,

$$H_{ij} v_{kj} \neq H_{ij} v_{lj} \quad i \neq k \neq l$$

(4)

Postulate 2 [11]: In a $K$ user system, since each receiver receives $K(K-1)$ interference components, in order to align $(K-1)$ interference signals along $K$ dimensions, the following condition needs to be satisfied.

$$\text{span}(H_{im} v_{km}) = \text{span}(H_{ln} v_{ln}) \quad k, l \neq i$$

(5)

C. Mathematical K - MIMO Model for $K=3$

The first step is to determine a set of span equations that will lead to interference alignment. These are formed by pairing the interference terms into equations, such that postulates 1 and 2 are satisfied. Table 1 shows a possible set of span equations for $K=3$. Each receiver will have $K(K-1)$ interference terms and $\frac{K(K-1)}{2}$ span equations. These equations will be used to solve for the precoders.

1) Solving for Precoders: Each receiver will apply $K$ precoders. From the span equations it is required to find a set of $K^2$ equations from which all the precoders can be computed. The equations must be ordered so that they form
TABLE I: Interference alignment conditions, 3-user network

<table>
<thead>
<tr>
<th>Rx 1</th>
<th>span((H_{11}v_{21})) = span((H_{12}v_{31})) span((H_{11}v_{31})) = span((H_{13}v_{33})) span((H_{12}v_{32})) = span((H_{13}v_{33}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rx 2</td>
<td>span((H_{21}v_{21})) = span((H_{23}v_{23})) span((H_{21}v_{23})) = span((H_{23}v_{23})) span((H_{22}v_{22})) = span((H_{23}v_{23}))</td>
</tr>
<tr>
<td>Rx 3</td>
<td>span((H_{31}v_{11})) = span((H_{33}v_{33})) span((H_{31}v_{31})) = span((H_{33}v_{33})) span((H_{32}v_{22})) = span((H_{33}v_{33}))</td>
</tr>
</tbody>
</table>

circular loop. This enables us to start by assuming a known value for \(v_{11}\), then go through all the precoders and end up back at \(v_{11}\). One possible set of equations for \(K = 3\) is shown below.

\[
\begin{align*}
v_{12} &= (H_{22})^{-1}H_{21}v_{11} & v_{33} &= (H_{13})^{-1}H_{11}v_{31} \\
v_{21} &= (H_{31})^{-1}H_{32}v_{12} & v_{32} &= (H_{22})^{-1}H_{23}v_{33} \\
v_{22} &= (H_{12})^{-1}H_{11}v_{21} & v_{23} &= (H_{13})^{-1}H_{12}v_{32} \\
v_{13} &= (H_{33})^{-1}H_{32}v_{22} & v_{11} &= (H_{31})^{-1}H_{33}v_{23} \\
v_{31} &= (H_{21})^{-1}H_{23}v_{13}
\end{align*}
\]

An initial value for \(v_{11}\) can be found by first defining a matrix \(E\) [11] as follows,

\[
E = (H_{31})^{-1}H_{32}(H_{13})^{-1}H_{12}(H_{22})^{-1}H_{23}(H_{13})^{-1}H_{11}
\]

\[\times (H_{21})^{-1}H_{23}(H_{33})^{-1}H_{32}(H_{12})^{-1}H_{11}(H_{31})^{-1}H_{32} \times (H_{22})^{-1}H_{21}
\]

The matrix \(E\) is obtained from (6) and \(v_{11}\) is arbitrarily chosen to be one of the eigen vectors of \(E\). Subsequently all the other precoders can be obtained in the order, \(v_{12}, v_{21}, v_{22}, v_{13}, v_{31}, v_{32}\) and \(v_{23}\).

2) Obtaining the Beamformer: The zero forcing beamformer matrix \(U_{i}\) is obtained by first defining a matrix \(H_{int}^{(1)}\) at each receiver and taking the Singular Value Decomposition (SVD) as follows,

\[
H_{int}^{(1)} = [H_{11}v_{21}] [H_{11}v_{31}] [H_{12}v_{32}]
\]

\[
= \left[\tilde{U}_{1}^{(1)} \ 0 \ 0 \ \tilde{V}_{1}^{(1)} \ 0 \ \tilde{V}_{1}^{(0)}\right]
\]

Where \(H_{int}^{(1)}\) is the set of aligned interfering column vectors at the first receiver. From (8) we can set \(\tilde{U}_{1}^{(0)} = U_{1}\), the zero forcing beamformer at the first receiver.

III. EXTENSION TO HIGHER ORDERS

In order to extend the \(K\)-MIMO model to higher dimensions, we propose a hypercube multi-point framework. This is shown in Figure 3. As in the \(K = 3\) case, the hypercube framework also consists of \(K\) transmitters and \(K\) receivers but the number of connections originating and ending at each node is \(log_{2}K\). This means that \(K\) has to be a power of 2. If we assign binary indices to each node in the network, as shown in Figure 3, each transmitter node will connect to a receiver node whose binary index differs by one bit.

In this model, there will be \(log_{2}K\) desired signals and \(log_{2}K(log_{2}K - 1)\) interfering signals at each receiver. Under this structure, instead of all to all connections between nodes, each transmit node in the network has \(log_{2}K\) connections originating from it and each receive node has \(log_{2}K\) terminating at it. This allows us to extend the interference suppression protocol to any value of \(K\) as long as \(K\) is a power of 2. The received signal at the \(i^{th}\) receiver is given by,

\[
y_{i} = \sum_{j=1}^{K} c_{ij}H_{ij}v_{kj}s_{kj} + \sum_{j=1}^{K} \sum_{k=1, k \neq i}^{K} c_{ij}H_{ij}v_{kj}s_{kj} + n_{i}
\]

where

\[
c_{ij} = \begin{cases} 
1 & \text{if the } j^{th} \text{ transmitter is connected to the } i^{th} \text{ receiver} \\
0 & \text{if the } j^{th} \text{ transmitter is not connected to the } i^{th} \text{ receiver}
\end{cases}
\]

The symbols \(s_{ij}\) corresponding to the links that are not connected are also set to 0.

A. Precoder Design

The length \(M\) precoders are picked from the \(K^{2}\) size twiddle factor matrix shown below. Each row of \(V\) corresponds to a precoder \(v_{ij}\), since they are linearly independent.

\[
V = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 \\
\omega^{1} & \omega^{2} & \omega^{3} & \ldots & \omega^{K-1} \\
1 & \omega^{2} & \omega^{3} & \ldots & \omega^{2K-1} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\omega^{K-1} & \omega^{2(K-1)} & \omega^{3(K-1)} & \ldots & \omega^{K-1}
\end{bmatrix}
\]
B. Obtaining the Beamformer

1) K = 4: Similar to the K = 3 case, the zero forcing beamformer matrix $U_i$ is obtained by first defining a matrix $H_{int}^{(1)}$ for each receiver $i$ and taking the SVD as follows:

$$H_{int}^{(1)} = \begin{bmatrix} H_{12}^{(1)} & H_{13}^{(1)} \\ H_{12}^{(1)} & H_{13}^{(1)} \end{bmatrix} = \begin{bmatrix} H_{12}^{(1)} & H_{13}^{(1)} \\ U_1^{(0)} & U_1^{(0)} \end{bmatrix} \begin{bmatrix} \bar{A}_1 & 0 \\ 0 & \bar{V}_1^{(0)} \end{bmatrix} H$$  \hspace{1cm} (12)

Where $H_{int}^{(1)}$ is the matrix that contains all the interfering column vectors at the first receiver. For a $K = 4$ hypercube network, there will be 2 interference terms at each receiver. In this respect, our $K > 3$ system differs from the above $K = 3$ model in which the $H_{int}^{(0)}$ contains only half of the interference terms since the two halves are aligned. In the $K > 3$ hypercube configuration, interference can be canceled purely by the use of zero forcing beamformers.

From (12) we can set $\bar{U}_1^{(0)} = U_1$, the zero forcing beamformer at the first receiver.

IV. ITERATIVE K–MIMO PROTOCOL WITH NOISY CHANNEL ESTIMATES

In this section we present an iterative framework for the K–user MIMO interference alignment, shown in Figure 4. The purpose of this is to account for the impact of channel estimation error and time varying channel coefficients on the interference cancellation. The iterative process is divided into two stages. In stage 1, we consider the initial channel estimation. In this stage, each transmitter transmits pilot signals to each receiver independently such that there is no interference.

The Cramér-Rao Lower Bound variance is given by,

$$\sigma_{ij-\text{CRLB}}^2 = \sigma_{ij-n}^2 tr\{D^{-1}\}$$ \hspace{1cm} (13)

where $\sigma_{ij-n}^2 = E[n_{ij}n_{ij}^T]$ is the variance of the noise $n_{ij}$. $D$ is a matrix whose elements are a function of the frequency bins for the pilot signals [16]. The initial channel estimation is performed using the Cramér-Rao variance given by equation (13). To simulate noisy channel estimates, we select error terms from Gaussian distribution of zero mean and variance equal to the Cramér-Rao variance in (13) and add them to the channel. This is given by, $H_{ij}^{\text{noisy}} = H_{ij} + \epsilon_{ij}$ where $\epsilon_{ij} \sim \mathcal{CN}(0, \sigma_{ij-\text{CRLB}}^2)$. Using the estimated channels, the precoders and beamformers are computed. Since the estimates are imperfect, the interference will be attenuated rather than completely canceled. We treat this residual interference as noise. In the second stage, all transmitters transmit to all receivers ($K = 3$) or $\log_2(K)$ receivers ($K > 3$). Using the residual interference from the previous stage, new channel estimates are made and new precoders and beamformers are computed. The iterative process is repeated until the residual interference at the end of each iteration reaches convergence.

A. Demodulator and Multi-User Capacity

The received signal with the beamformer applied, takes the following form,

$$y_{ij}(\hat{f}, \hat{\gamma}) + T_{ij}(\hat{f}, \hat{\gamma}) + n_{ij}(\hat{f}, \hat{\gamma}) = U_i^H H_{ij} x_{ij}(\hat{f}, \hat{\gamma}) + \epsilon_{ij}(\hat{f}, \hat{\gamma})$$  \hspace{1cm} (14)

where $A_{ij}(\hat{f}, \hat{\gamma}) = \sqrt{M \frac{1}{P_j}}$, $P_j$ is the transmit power at the $j^{th}$ transmitter, $L_{ij}$ is the path loss across distance $d_{ij}$ from the $j^{th}$ transmitter to the $i^{th}$ receiver and $n_{ij}(\hat{f}, \hat{\gamma})$ is the noise at the $i^{th}$ receiver in frequency bin $\Delta_f$.

As part of the demodulation, we define $\Gamma_{ij}(\hat{f}, \hat{\gamma}) = U_i^H H_{ij}$ and then find the SVD as follows,

$$\Gamma_{ij}(\hat{f}, \hat{\gamma}) = U_i^H H_{ij}$$

where $\Gamma_{ij}(\hat{f}, \hat{\gamma})$ has dimensions $K \times M$, $\Phi_{ij}$ is the left singular matrix of dimensions $K \times M$, $\Psi_{ij}$ is the right singular matrix of dimensions $M \times M$ and $\Lambda_{ij}$ is a $K \times M$ diagonal matrix with singular values along the diagonal. At frequency bin $\Delta_f$,

$$\Lambda_{ij} = \begin{bmatrix} \lambda_{ij}(1) & 0 & \ldots & 0 & 0 \\ 0 & \lambda_{ij}(2) & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \lambda_{ij}(M) & \ldots & 0 \end{bmatrix}$$ \hspace{1cm} (15)

We define the estimated information signal from transmitter $j$ to receiver $i$ as,

$$\hat{s}_{ij}(\hat{f}, \hat{\gamma}) = [\hat{s}_{ij}(\hat{f}, \hat{\gamma}), \ldots, \hat{s}_{ij}(\hat{f}, \hat{\gamma}, K)]^T$$ \hspace{1cm} (17)

The detected signal is given by,

$$\hat{s}_{ij}(\hat{f}, \hat{\gamma}) = \sum_{\gamma=1}^{M} \hat{s}_{ij}(\hat{f}, \hat{\gamma} \cdot (\hat{s}_{ij}(\hat{f}, \hat{\gamma}))^{-1}$$ \hspace{1cm} (18)
where,
\[
[s_{ij0\Delta_f}(1), \ldots, s_{ij0\Delta_f}(\gamma), \ldots, s_{ij0\Delta_f}(K)]^T := \Phi_{ij}^H \cdot (\overline{y}_{ij\Delta_f} + \overline{I}_{ij\Delta_f} + \overline{n}_{ij\Delta_f})
\]
(19)
and
\[
[s_{ij1\Delta_f}(1), \ldots, s_{ij1\Delta_f}(\gamma), \ldots, s_{ij1\Delta_f}(K)]^T := \Psi_{ij}^H \cdot \hat{v}_{ij}
\]
(20)

In (20) we assume that \(\hat{v}_{ij} \approx v_{ij}\). Further, we assume that due to the interference cancellation, \(\overline{I}_{ij\Delta_f} < < \overline{n}_{ij\Delta_f}\). Therefore we can rewrite (19) as below,
\[
\Phi_{ij}^H \cdot (\overline{y}_{ij\Delta_f} + \overline{I}_{ij\Delta_f} + \overline{n}_{ij\Delta_f}) \approx \Phi_{ij}^H \cdot (\overline{y}_{ij\Delta_f} + \overline{n}_{ij\Delta_f})
\]
(21)

The demodulated signal obtained from (18) takes the form,
\[
\hat{s}_{ij\Delta_f} = \sum_{\gamma=1}^{M} \lambda_{ij\Delta_f}(\gamma) \hat{s}_{ij\Delta_f}(\gamma) + \hat{n}_{ij\Delta_f}
\]
(22)
where \(\lambda_{ij\Delta_f}(\gamma)\) is the \(\gamma^{th}\) diagonal element in \(\Lambda_{ij}\) and \(\hat{n}_{ij\Delta_f}\) is the noise after demodulation.

The SINR after interference suppression and demodulation can be approximated as SNR, \(\rho_{ij}\) and is computed as follows,
\[
\rho_{ij} = \frac{\left| \sum_{\gamma=1}^{M} \lambda_{ij\Delta_f}(\gamma) \hat{s}_{ij\Delta_f}(\gamma) \right|^2}{|\hat{n}_{ij\Delta_f}|^2}
\]
(23)

1) Multi-User Capacity for \(K = 3\) User MIMO: The multi-user capacity in bits/sec for the \(K\)-MIMO system is given by,
\[
C_{\text{bits/sec}} = \frac{W}{K} \sum_{i=1}^{K} \sum_{j=1}^{K} \log_2(1 + \rho_{ij})
\]
(24)
where \(W\) is the bandwidth in Hz and \(\rho_{ij}\) is the signal to noise ratio of the signal between the \(j^{th}\) transmitter and the \(i^{th}\) receiver.

2) Multi-User Capacity for Extended \(K > 3\) User MIMO: The multi-user capacity in bits/sec for the hypercube system is,
\[
C_{\text{bits/sec}} = \frac{W}{K} \sum_{i=1}^{K} \sum_{j=1}^{\log_2 K} \log_2(1 + \rho_{ij})
\]
(25)
where \(W\) is the bandwidth in Hz and \(\rho_{ij}\) is the signal to noise ratio of the signal between the \(j^{th}\) transmitter and the \(i^{th}\) receiver.

3) Multi-User Capacity for Realistic SINR Gains: The singular values in (16) represent the gains experienced by the transmitted symbols after the beamformer is applied. Figure 5 shows relative frequency distributions of the powers of the singular values multiple trials in which different channel coefficients were selected and the singular value decomposition in (15) was performed.

It can be observed that for \(K > 3\) systems, the average power gain of the singular values can be approximated by \((K - 1)^2\). The number of singular values is \(\frac{M}{2}\) as shown in (16). Assuming that each receiver signal experiences the average gain, we obtain a closed form expression for the Shannon capacity for \(K > 3\) hypercube networks as shown below,
\[
C_{\text{bits/sec}} \approx \frac{W}{K} \cdot K \cdot \log_2 K \cdot \log_2(1 + (K - 1)^2) \cdot \frac{M}{2} \cdot \frac{S_b}{N_b + I_a}
\]
(26)
Where \(W\) is the bandwidth and \(\frac{S_b}{N_b + I_a}\) is the signal to noise plus interference ratio which uses the interference after suppression (refer Figure 2) and \(\frac{S_b}{N_b + I_a} = (K - 1)^2 \cdot \frac{M}{2} \cdot \frac{S_b}{N_b + I_a}\). Figure 6 shows the variations of spectral efficiency from (26) as \(M\) increases.

Fig. 5: Relative frequency distributions of singular value power gains (15) for \(K = 3, 4, 8\) and 16.

V. 5G DENSE NETWORK SCENARIO
An example of a 5G dense network scenario is shown in Figure 7. We consider an network model in which larger eNodeBs are communicating with smaller low power Remote Radio Heads (RRH). Small cells formed by large numbers

![Figure 7](image.png)

![Figure 6](image.png)
of RRHs on rooftops or on street lamps forming a dense 5G network could be a possible use case for this model. Without loss of generality, we assume in our simulations, that the eNodeBs are the transmitting nodes and the RRHs are the receiving nodes. The same model can also be applied for communications between RRHs and User Equipments (UE).

Fig. 7: Example dense network scenario showing $K$-user MIMO connections between eNodeBs and Remote Radio Heads.

Fig. 8: Cumulative distribution function of spectral efficiency in bits/sec/Hz for $K = 3$. Multiple iterations were run and 3 users were selected such that the 9 $K$-user MIMO connections had the best singular value gains.

Fig. 9: Cumulative distribution function of spectral efficiency in bits/sec/Hz for $K = 4, 8, 16$. Multiple iterations were run and $K$ users were selected such that the $K\log_2 K$ $K$-user MIMO connections had the best singular value gains.

where $\mu = (K - 1)^2$ is the mean of the distribution.

A. Scheduling Protocol

We assume a scenario in which there are $T_{max}(\geq K)$ transmitters and $R_{max}(\geq K)$ receivers. Since the $K$-user MIMO protocol requires that the number of transmitters be equal to the number of receivers, different groups of receivers have to be served in different time slots. The protocol is shown in Algorithm 1. The goal is to achieve the maximum possible capacity by opportunistic selection of receivers to serve in each time slot based on the singular value gains in (15).

In each time slot, $K$ receivers are selected. The receivers associate with the best transmitters based on a minimum distance criterion. For $K < 6$, transmitters are located on the vertices of the hexagonal cell. For $K > 6$, transmitters are assumed to be located on a circumcircle to a hexagonal cell. $K$ out of $T_{max}$ transmitters are chosen and distances are measured. The combination of $K$ transmitters that has the smallest maximum distance is chosen. This maximizes the receive SNR to a certain degree by minimizing the path loss.

VI. SCHEDULING PROTOCOL FOR DENSE MASSIVE MIMO NETWORKS USING INTERFERENCE ALIGNMENT

Equation (26) assumes that each of the $K\log_2 K$ signals experiences the average gain. But it is clear from Figure 5 that there is considerable variation in the power gains above and below the mean. So in this section, we present a scheduling protocol that uses opportunistic selection to schedule $K$ receivers in each time slot on channels that have the maximum gains.

We approximate the distributions in Figure 5 to be exponential with the means shown. The gain, $g_{ij}$ experienced by a signal from the $j^{th}$ transmitter to the $i^{th}$ receiver after applying the beamformer is given by,

$$g_{ij} \sim \text{Exp}(\mu)$$

(27)
To further improve the SNR, the $K$ users that experience the best channel conditions in the given time slot are scheduled. For simulation purposes, we draw $R_{\text{max}}$ gains from an exponential distribution of appropriate mean (based on $K$) such that the $K$–user MIMO connections experience the top $K^2$ (for $K = 3$) out of $R_{\text{max}}$ singular value gains and $K\log_2 K$ (for $K > 3$). This is repeated for multiple iterations (time slots) and the $K$–user spectral efficiency is computed in each time slot. The results are plotted in Figures 8 and 9. The capacity equation used in Figure 8 for $K = 3$ is,

$$C_{\text{bits/sec}} = \frac{W}{K} \sum_{i=1}^{K} \sum_{j=1}^{K} \log_2(1 + g_{ij} \cdot \frac{M}{2} \cdot \frac{S_b}{N_b + I_a})$$ (28)

where $W$ is the bandwidth in Hz and $g_{ij}$ is the singular value gain imparted to the signal from the $j^{th}$ transmitter to the $i^{th}$ receiver.

And for $K > 3$ in Figure 9,

$$C_{\text{bits/sec}} = \frac{W}{K} \sum_{i=1}^{K} \log_2(1 + g_{ij} \cdot \frac{M}{2} \cdot \frac{S_b}{N_b + I_a})$$ (29)

where $W$ is the bandwidth in Hz and $g_{ij}$ is the singular value gain imparted to the signal from the $j^{th}$ transmitter to the $i^{th}$ receiver. Some parameters used in the simulations include, transmit power of 46dBm, thermal noise power of $-204$dBm, noise figure of 4dB. The path loss model used is given by $L_{ij} = k - 10\alpha \log_{10}(\frac{d_{ij}}{d_0})$ where, $k$ is the path loss constant in dB, $\alpha$ is the path loss exponent, $d_{ij}$ is the distance between the $i^{th}$ transmitter and the $j^{th}$ receiver in meters and $d_0$ is the reference distance in meters. $k$ of 0dB, $\alpha$ of 3 and $d_0$ of 1m were used.

Algorithm 1: Scheduling protocol for 5G dense networks using interference alignment

| Input | Set of transmitters, $T$, $|T| = T_{\text{max}}$ |
| Input | Set of receivers, $R$, $|R| = R_{\text{max}}$ |
| Input | Set of channel gains, $G$ (27) |

1. $n = 0$;
2. max iterations (time slots) = $N$;
3. while $n \leq N$ do
4. Select $K$ receivers from $R$;
5. Associate receivers with $K$ transmitters from $T$ by minimizing maximum distance;
6. Generate hypercube connections with top $K\log_2 K$ singular value gains from $G$;
7. Compute multi-user capacity, $C(n)$ (28), (29);
8. $n = n + 1$;
9. end

VII. CONCLUSION

In this paper, we have presented a 5G dense network design methodology for networks that have employed interference alignment. We have provided a modified $K$–user model that can be extended beyond $K = 3$. We have generated simulation results under realistic Cramér-Rao bound channel estimation error constraints. In addition, we have presented opportunistic user scheduling protocols based on the best singular value gains of the channel. Future work on the interference alignment protocol includes developing a software application that could be integrated into the Cloud-Radio Access Network (C-RAN) framework for 5G dense networks. Learning algorithms can be applied for better design of precoders and beamformers for optimum gain and interference cancellation for $K > 3$ systems.

A. Acknowledgment

The authors would like to acknowledge the support from Air Force Research Laboratory and OSD for sponsoring this research under Testing, Evaluation, and Control of Heterogeneous Large Scale systems of Autonomous Vehicles (TECHLAV) agreement number FA8750-15-2-0116.

REFERENCES