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Analysis of the Effects of Communication Delays for Consensus of Networked Multi-agent Systems

Myrielle Allen-Prince, Christopher Thomas, and Sun Yi*

Abstract: Achieving cooperation and coordination in a network of multi-agent systems is key to solving the consensus problem. Synchronization of such systems requires consistent communication between agents to reach a consensus, which is not feasible in the presence of delays, data loss, disturbances, and other unpredictable factors. Communication delays combined with environmental uncertainties can cause adverse effects and negatively change the behavior of the networked system preventing synchronization. In this paper, the stability of the delayed networked systems is analyzed using delay differential equations. Solving these equations has not been feasible, because of the infinite number of characteristic roots. The approach based on the Lambert W function has the capability of analytically solving delay differential equations. The approach is used to quantify and analyze the stability of the delayed networked systems. The communication between the agents is modeled using the graph theory and the Laplacian matrix. The stability is analyzed by incorporating the Laplacian matrix into the Lambert W function based approach which provides the locus of the eigenvalues of the system as delay changes. Sensitivities and convergence speed with respect to delay for various topologies of the network are presented for comparison. The numerical results and implementation using MATLAB/Simulink are presented for illustration.

Keywords: Consensus, delay, multiple-agent systems, sensitivity, stability, topology.

1. INTRODUCTION

Coordinated and cooperative control has been studied in [1,2] to develop control algorithms for multi-agent systems (MAS). Compared to a single-agent system, MAS can perform multiple complex tasks in less time and encountering less setbacks or task failures [3,4]. MAS can improve security, search and rescues, environmental monitoring, and much more. The consensus problem consists of making the individual agent within a network come to an agreement on a decision and/or task. This gives these multi-agents the ability to perform cooperatively in a coordinated manner. Some applications of the consensus problem are formation flight, traffic monitoring and control, swarming/flocking, satellite formation, and many more systems that can communicate and contain cooperative control capabilities. There is a great potential but there are also many challenges that must be dealt with and overcome. Networked systems are prone to failures due to uncertainties such as disturbances, nonlinearity, information signal loss, and communication delay. This research focuses on analyzing the stability, convergence speed, and sensitivity of networks of multi-agent systems in the presence of delay.

Delays in the network can lead to instability and unpredictable behaviors of the system and its agents. These agents are expected to perform designated tasks requiring them to communicate. This communication is done wirelessly, which is not always reliable. Wireless communication has major challenges within itself such as signal fading, interference, attenuation, intermittent connectivity, or link breakage. All of which can lead to delayed communication between agents. The effects of delay are neither trivial nor predictable [5]. One stipulation of the consensus problem is synchronization between agents that can be greatly affected by the delay. Time delay and communication topology are the key factors that influence the stability of the multi-agent system [2]. To ensure the agents reach a consensus, we need to know when and how delay will affect it. Analysis of the delay's effects is challenging. Delay in the characteristic equation can be represented by exponential functions, which lead to an infinite spectrum. Thus, when modeling a delayed system using delay differential equations (DDEs) the delay term introduces an

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Myrielle Allen-Prince, Christopher Thomas, and Sun Yi are with Department of Mechanical Engineering in North Carolina A and T State University, 1601 East Market Street, Greensboro, North Carolina 27411, USA (e-mails: myrielle.allenprince@gmail.com, cothomas@aggies.ncat.edu, syi@ncat.edu).
* Corresponding author.

infinite number of characteristic roots and, thus, solving these equations has not been feasible. In this paper, the stability of the delayed networked systems is analyzed using an approach based on the Lambert W function, which has the capability of analytically solving delay differential equations. The approach is used to quantitatively analyze the stability of the delayed networked systems. The communication between the agents is modeled using the graph theory and the Laplacian matrix. The stability is analyzed by incorporating the Laplacian matrix into the Lambert W function based approach. That enables one to obtain the locus of the eigenvalues of the system as delay changes. Also, sensitivities and convergence speed with respect to delay for various topologies of the network are presented for comparison. The numerical results and application of a testbed using MATLAB/Simulink are presented for illustration.

This is organized as follows: Section 2 contains background information of this research on the consensus problem of multi-agent systems The focus, challenges to overcome, and the methods used to address these challenges are described. Section 3 provides the methodology used to model these systems and analyze their behaviors. A new method is used to analyze the systems stability and sensitivity to ensure its synchronization ability. Section 4 contains validation of the methods used in Section 3. Section 5 provides a summary of the results and recommendations for future work.

2. CONSENSUS AND DELAY

2.1. Consensus

The consensus problem has been integrated and applied in many fields of study for networks of dynamic systems. The increased attraction to the distributed coordination of these systems is partially due to the broad applications of MAS such as cooperative control of unmanned aerial vehicles (UAVs), formation control, flocking, distributed sensor networks, satellite clustering, and congestion control in communication networks [6]. In all these applications, MAS are groups of agents that need to come to an agreement on a task. There are multiple studies on the consensus problem [1, 2], most of which focused on switching topology and time delay [2, 6] and cooperative and coordinated control [1, 7–9].

2.2. Time delay

Time delay is present typically in systems that are required to communicate over a network or channel [10]. This is due to many factors, a few being the signal transmission speed and network congestion [11, 12]. Ignoring time delay is impossible, because it is always present and can cause instability in the system [11]. Delay in the network can lead to unpredictable behaviors of the system and its agents. The delay provides the model with an infinite number of characteristic roots creating difficulty in determining the stability or designing controllers.

Lyapunov and linear matrix inequality (a.k.a. LMI) methods, robust controller synthesis has been used to obtain a maximal allowable upper bounds for known and unknown delays [11, 13, 14]. Many types of Lyapunov functions have been used to handle delays [15, 16]. The latter has conservative results, which applies to time-varying delays without restriction but boundedness and Krasovskii-Lyapunov is a classical technique and requires a bounded derivative [5]. Refer to [5] for an article on multiple time delay techniques and open problems. The Lambert W Function-based approach provides solutions to a type of DDEs and the stability by only using its principle branch [17].

2.3. Lambert W function-based approach

The Lambert W Function is defined as

$$z = W_k(z)e^{W_k(z)}, \tag{1}$$

where z is any complex number, $W(z)$ is the Lambert W function. Also, $k = -\infty, \cdots, +\infty$ represent the branches of the W function. These branches correspond to the infinite number of solutions the Lambert W function can provide. The principal branch is $W_0(z)$ is the only required branch to determine stability in the case of infinite roots [17–19]. This function can handle the exponential term in the characteristic equation of DDEs. For detailed information on the Lambert W function, refer to [18]. MATLAB has a Lambert W function embedded, which was used for calculations within this research.

2.4. Graph theory and Laplacian matrix

Graph theory is used in mathematics and computer science, where graphs are used to show the connection or relationship between a specific group of objects. These graphs typically consist of circles, lines and arrows. The circles are called nodes or vertices, which are the objects (agents) within the group. The lines or arrows are called edges, which indicate the connected/communicating nodes. Graphs can be categorized as undirected, which are represented by lines or directed, which are represented arrows. This research uses directed graphs to indicate the information being sent or received, or the direction of communication. A directed graph also known as a topology is defined as $G = (V, E)$, where V is a set of nodes or vertices $\{v_1, v_2, ..., v_n\}$, and E is a set of edges. An edge can be defined as $e_{ij} = (v_i, v_j)$, where v_i is the node sending and v_j is receiving. Some sources define a graph as $G = (V, E, A)$, where A is the adjacency matrix $[a_{ij}]$ which, likes edges, shows which nodes are connected.

A Laplacian Matrix is developed from a graph (topology). These matrices consist of mostly 0 and -1 on the off-diagonal. The diagonal will depend on the number